HYDRAULIC TURBINES

CENTRIFUGAL PUMPS

BY

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PREFACE TO THE THIRD EDITION

Since this book was first written, practice has changed to such an extent that many statements, which were true at that time, are not true today. These portions have been entirely rewritten so as to present the very latest features in construction and practice. Also practically every other chapter has been altered and new matter and illustrations inserted, where it was thought that greater clearness could be so attained.

The presentation of the theory has been quite carefully considered and has been largely rewritten in order to be more effective. An attempt has been made to so arrange this that the fundamental principles could be grasped without going into a lot of technical details. If desired, certain portions of the latter, of theoretic interest only, can be omitted without breaking the continuity of thought.

Chapters have been added on turbine governors and on the methods of turbine design. The latter has been inserted in order to meet a demand for something on that subject. The methods that are given are those employed by the best designers at the present time. The procedure avoids the old "cut and try" practice on the one hand, as well as a highly theoretical treatment, that is of no practical value, on the other. It is rather a happy compromise between the two. The author is still of the opinion that the greater number of engineers are concerned with the construction and operating characteristics of turbines rather than with the details of their design. But there are some phases of turbine performance and construction that can be understood more completely, if approached from the view point of hydraulic design.

Questions and numerical problems have been added at the end of every chapter, in order to increase the usefulness of the book for instruction purposes. The questions are intended to call attention to the most important features presented in the chapter and also to bring out more clearly the thought there

to suit his individual course, and to vary them from year to year, rather than to incorporate such in the book.

The notation has been changed slightly in the present edit in order to conform more closely to the standard notat recommended by the Society for the Promotion of Engineer Education.

The author is indebted to many teachers and students, v have used the former editions, and also to engineers with wh he has discussed these matters for numerous suggestions wh have been helpful to him in the preparation of the present volu

R. L. D

PASADENA, CALIF.,
February, 1920.

PREFACE TO THE SECOND EDITION

In addition to correcting typographical errors and rewritin two articles, the issuing of a second edition has afforded a opportunity to add new material which it is believed will increas the sphere of usefulness of the book. The discussion of severa matters in the text has been amplified and there have been adde numerous questions and problems. This together with the 1 tables of test data in Appendix C will afford much suitabl material for instruction purposes.

The author wishes to acknowledge his indebtedness to Pro E. H. Wood of Cornell University for his careful criticism of th first edition and to Prof. W. F. Durand of Leland Stanfor University for much valuable assistance.

R. L. D.

ITHACA, N. Y.,
August, 1914.

PREFACE TO THE FIRST EDITION

The design of hydraulic turbines is a highly specialize industry, requiring considerable empirical knowledge, whicl can be aquired only through experience; but it is a subject i which comparatively few men are interested, as a relativel small number are called upon to design turbines. But with th increasing use of water power many men will find it necessar to become familiar with the construction of turbines, understan their characteristics, and be able to make an intelligent selectio of a type and size of turbine for any given set of conditions To this latter class this book is largely directed. However, clear understanding of the theory, as here presented, ought t be of interest to many designers, since it is desirable that Ameri can designs be based more upon a mathematical analysis, as i Europe, and less upon the old cut and try methods.

The broad problem of the development of water power i treated in a very general way so that the reader may understan the conditions that bear upon the choice of a turbine. Thus th very important items of stream gauging and rating, rainfall an runoff, storage, etc., are treated very briefly, the detailed stud of these topics being left for other works.

The purpose of the text is to give the following: A genera idea of water-power development and conditions affecting th turbine operation, a knowledge of the principal features o construction of modern turbines, an outline of the theory an the characteristics of the principal types, commercial constants means of selection of type and size of turbine, cost of turbine and water power and comparison with cost of steam power A chapter on centrifugal pumps is also added. It is hoped tha the book may prove of value both to the student as a text an to the practicing engineer as a reference. R. L. D.

CORNELL UNIVERSITY, ITHACA, N. Y.,
August, 1913.

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= total area of streams in square feet measured normal to absolute velocity.
= total area of streams in square feet measured normal to relative velocity.
= height of turbine runner in inches.
= coefficient of discharge in general.
= $V_1/\sqrt{2gh}$.
= coefficient of contraction.
= coefficient of velocity.
= coefficient of radial velocity.
= coefficient of tangential velocity.
= diameter of turbine runner in inches.
= efficiency.
= hydraulic efficiency.
= mechanical efficiency.
= volumetric efficiency.
= force in pounds.
= friction factor.
= acceleration of gravity in feet per second per second.
= total effective head $= z + V^2/2g + p/w$.
= any loss in head in feet.
= head in feet.
= head lost in friction in turbine or pump.
= head converted into mechanical work or vice versa.
= any factor.
= capacity factor.
= power factor.
= any coefficient of loss.
= revolutions per minute.
= speed for maximum efficiency.
= specific speed $= N_e\sqrt{\text{B.h.p.}}/h^{5/4}$.
= abstract number.
= abstract number.
= axis of rotation.
= power.
= intensity of pressure in pounds per square foot.
= total quantity in cubic feet.
= rate of discharge in cubic feet per second.

V_u = tangential component of absolute velocity = $V \cos \alpha$.
v = velocity of water relative to wheel in feet per second.
W = pounds of water per second = wq.
w = density of water in pounds per cubic foot.
$x = r_2/r_1$.
$y = A_1/a_2$.
α = angle between V and u (measured between positive directions).
β = angle between v and u (measured between positive directions).
ϕ = ratio $u_1/\sqrt{2gh}$.
ϕ_e = value of ϕ for maximum efficiency.
ω = angular velocity = u/r.

The subscript (1) refers to the point of entrance and the subscript (2) refers to the point of outflow in every case.

HYDRAULIC TURBINES

CHAPTER I

INTRODUCTION

1. Historical.—Water power was utilized many centuries o in China, Egypt, and Assyria. The earliest type of water eel was a crude form of the current wheel, the vanes of which oped down into the stream and were acted upon by the impact the current (Fig. 1). A large wheel of this type was used pump the water supply of London about 1581. Such a wheel uld utilize but a small per cent. of the available energy of e stream. The current wheel, while very inefficient and ited in its scope, is well suited for certain purposes and is t yet obsolete. It is still in use in parts of the United States,

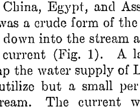


FIG. 1.—Current wheel.

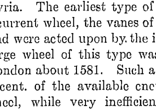


FIG. 2.—Breast wheel.

China, and elsewhere for pumping small quantities of water irrigation.

The undershot water wheel was produced from the current eel by confining the channel so that the water could not escape der or around the ends of the vanes. This form of wheel was pable of an efficiency of 30 per cent. and was in wide use up to out 1800.

The overshot water wheel (Fig. 3) also utilized the weight of the water. When properly constructed it is capable of an efficiency of between 70 and 90 per cent. which is as good as the modern turbine. The overshot water wheel was extensively used up to 1850 when it began to be replaced by the turbine, but it is still used as it is well fitted for some conditions.

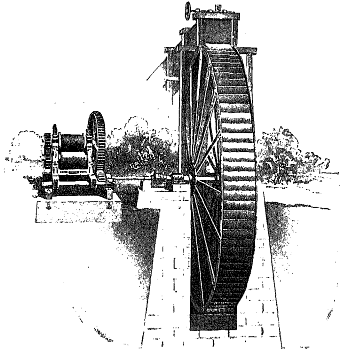


FIG. 3.—I. X. L. steel overshot water wheel. (*Made by Fitz Water Wheel Co.*)

2. The Turbine.—The turbine will be more completely described in a later chapter but in brief it operates as follows: A set of stationary guide vanes direct the water flowing into

may thus be applied to the turbine, since the latter is a special type of water wheel, according to the definition in the preceding paragraph. Second it may be used to designate the types of machines described in Art. 1 in order to distinguish them from

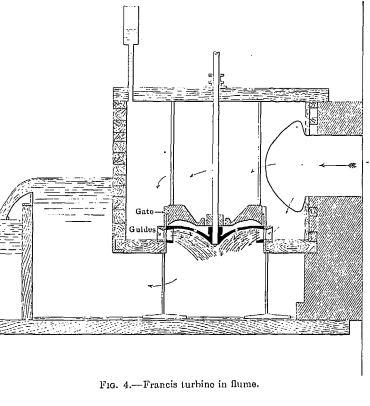


Fig. 4.—Francis turbine in flume.

the modern turbine. Third it may be understood to indicate impulse turbines of the Pelton type as contrasted with turbines of the reaction type. In this book the term is used in the first or second sense only, the context making it clear which is meant

1. The latter occupies smaller space.
2. A higher speed may be obtained.
3. A wider range of speeds is possible.

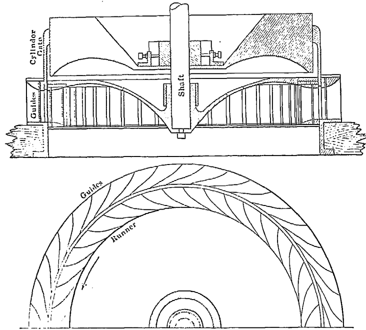


FIG. 5.—Francis turbine.

4. It can be used under a wide range of head, whereas the head for an overshot wheel should be only a little more than the diameter of the wheel.

5. A greater capacity may be obtained without excessive size.

6. It can work submerged.

7. There is less trouble with ice.

8. It is usually cheaper.

inefficient and unsatisfactory. In such cases the overshot water wheel may be better. The latter has a very high efficiency when the water supply is much less than its normal value. It is adapted for heads which range from 10 to 40 ft. and for quantities of water from 2 to 30 cu. ft. per second.[1]

An overshot wheel on the Isle of Man is 72 ft. in diameter and develops 150 h.p. Another at Troy, N. Y., was 62 ft. in

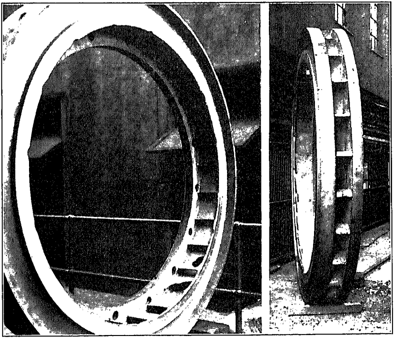


Fig. 6. —Pure radial inward flow runner of the original Francis type.

diameter, 22 ft. wide, weighed 230 tons, and developed 550 h.p. The latter is now in a state of ruin.

5. Essentials of a Water-power Plant.—A water-power plant

be no more than a diversion wall to deflect a portion of the current into the intake.

3. *Intake Equipment.*—This usually consists of racks or screens to keep trash from being carried down to the wheels and of head gates so that the water may be shut off, if need be.

4. *The Conduit.*—The water may be conducted by means of an open channel called a canal or flume, or through a tunnel, or by means of a closed pipe under pressure, which is called a penstock if it leads direct to the turbines.

5. *The Forebay.*—A small equalizing reservoir is often placed at the end of the conduit from the main intake and the water is then led from this to the turbines through the penstock. This is called the forebay and is also referred to as the headwater. In the case of a plant without any storage reservoir the body of water at the intake is often termed the forebay.

6. *The Turbine.*—The turbine with its case or pit and draft tube, if any, comprise the setting.

7. *The Tail Race.*—The body of water into which the turbine discharges is called the tail water. The channel conducting the water away is the tail race.

6. QUESTIONS

1. What is a turbine? What is a water wheel?

2. Under what circumstances would a current wheel be used? Could a turbine be used under the same conditions? What is the advantage of the undershot wheel over the current wheel?

3. Under what circumstances would an overshot water wheel be used? Could a turbine be used under the same conditions? Could an overshot wheel replace any turbine?

4. What elements would be found in every water-power plant? What elements may be in some and lacking in others? What is the difference between a storage reservoir and a forebay?

CHAPTER II

TYPES OF TURBINES AND SETTINGS

7. Classification of Turbines.—Turbines are classified according to:

1. Action of Water
   - (*a*) Impulse (or pressureless).
   - (*b*) Reaction (or pressure).
2. Direction of Flow
   - (*a*) Radial outward
   - (*b*) Radial inward
   - (*c*) Axial (or parallel)
   - (*d*) Mixed (radial inward and axial).
3. Position of Shaft
   - (*a*) Vertical.
   - (*b*) Horizontal.

8. Action of Water.—In the impulse turbine the wheel passages are never completely filled with water. Throughout the flow the water is under atmospheric pressure. The energy of the water leaving the stationary guides and entering the runner is all kinetic. During flow through the wheel the absolute velocity of the water is reduced as the water gives up its kinetic energy to the wheel. In Europe a type of impulse turbine commonly used is called the Girard turbine. In the United States practically the only impulse turbine is the tangential water wheel or impulse wheel, more commonly known as the Pelton wheel. (See Fig. 7.)

In the reaction turbine the wheel passages are completely filled with water under a pressure which varies throughout the flow. The energy of the water leaving the stationary guide vanes and entering the runner is partly pressure energy and partly kinetic energy.[1] During flow through the wheel both the pressure and the absolute velocity of the water are reduced as the water gives up its energy to the wheel.

dynamic force is due to a change produced in the velocity of the water and the distinction is largely artificial. And in modern turbines the so-called "impulse" at entrance and "reaction" at outflow may be effective in either type.

A far better classification is as pressureless and pressure turbines. Another classification is as partial and complete admission turbines, as in the former type the water is admitted

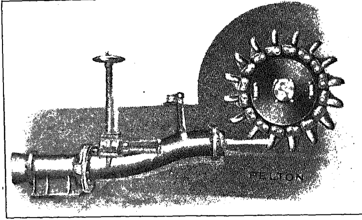


FIG. 7.—Tangential water wheel with deflecting nozzle.

at only a portion of the circumference while in the latter type is necessarily admitted around the entire circumference.

9. Direction of Flow.—Radial flow means that the path of particle of water as it flows through the runner lies in a plan which is perpendicular to the axis of rotation. If the water ente at the inner circumference of the runner and discharges at th outer circumference we have an outward flow type known the Fourneyron turbine. (See Fig. 78.)

If the water enters at the outer circumference of the runn and discharges at the inner circumference we have an inwa flow type as in the original Francis turbine shown in Figs.

ome extent in Europe.

If the water enters a wheel radially inward and then during s flow through the runner turns and discharges axially we ave a mixed flow turbine. This is known as the American type f turbine and is also called a Francis turbine, though it is not dentical with the one built by Francis.

Modern reaction turbines are practically all inward flow urbines of the mixed flow type and to this type our discussion vill be confined.

10. Position of Shaft.—The distinction as to position of shaft is bvious. The vertical shaft turbines are, however, further classied as right-hand or left-hand turbines according to the direction f rotation. If, in looking down upon the wheel from above, the otation appears clockwise it is called a right-hand turbine. The everse of this is a left-hand turbine.

So far as efficiency of the runner alone is concerned there is ttle difference between vertical and horizontal turbines. Other hings being equal, the hydraulic losses should be identical in ither case, but there might be some difference in the friction of ne bearings. As the latter is only a relatively small item, a easonable variation in its value would have but slight effect n the efficiency.

But when we consider the runner and draft tube together, we nay find a difference, since the draft tubes are not necessarily qually efficient in the two cases. The single-runner, verticalhaft turbine, as shown in Fig. 9, is readily seen to lend itself to a nore efficient draft tube construction than the horizontal-shaft nit, as shown in Fig. 11, with the necessary sharp quarter turn ear the runner where the velocity of the water is still high. f the velocity of discharge from the runner is low, the difference n the two cases may be insignificant, but, where the velocity of ne water is relatively high, the draft tube for the vertical-shaft vheel may be decidedly better.

In general a horizontal shaft is more desirable from the stand-

are usually met with in low-head plants, it will be found that ordinarily the vertical setting is used only for low heads. (See Fig. 8.)

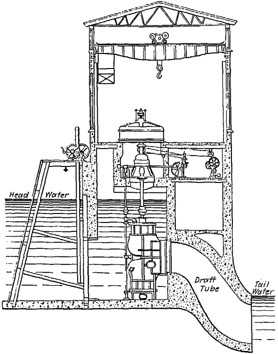


FIG. 8.—Pair of vertical shaft turbines in open flume

The horizontal shaft turbine is used where the turbine can be set above the tail water level and if the generator or other machinery that it drives can be set at the same elevation. This is almost always the case with a high-head plant and is also quite frequently the case with a low-head plant. (See Fig. 10.)

These statements are purely general and there are many exceptions.

number, and

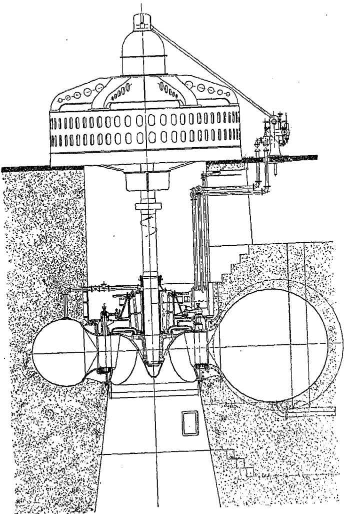


FIG. 9.—Vertical shaft turbine with spiral case.

The generator is mounted between the two bearings and the
bine runner, which is relatively light, is overhung on the e
the generator shaft. Sometimes there are two runners fo
generator and in this case one may be overhung on either
The former is called the single-overhung and the latte
double-overhung construction.

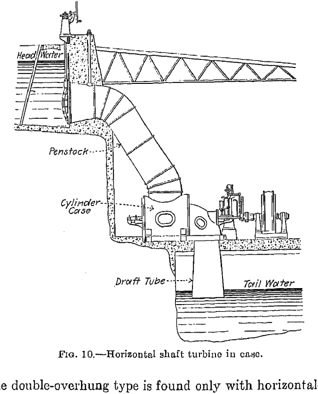


FIG. 10.—Horizontal shaft turbine in case.

The double-overhung type is found only with horizontal
units and naturally requires two separate cases and two
tubes. On the other hand with either a horizontal or ve
shaft we may have two runners discharge into a common

runner of either the single or double discharge type or two runners with separate draft tubes.

12. The Draft Tube.—Occasionally reaction turbines have been set so as to discharge above the tail water; in such cases the

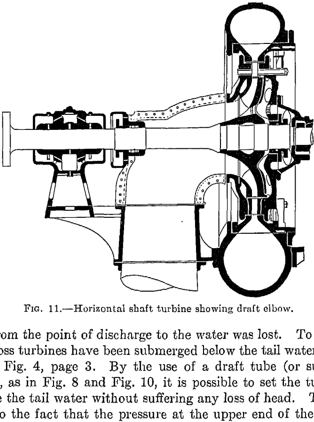


FIG. 11.—Horizontal shaft turbine showing draft elbow.

fall from the point of discharge to the water was lost. To avoid this loss turbines have been submerged below the tail water level as in Fig. 4, page 3. By the use of a draft tube (or suction tube), as in Fig. 8 and Fig. 10, it is possible to set the turbine above the tail water without suffering any loss of head. This is due to the fact that the pressure at the upper end of the draft tube is less than the atmospheric pressure. This suction compensates for the loss of pressure at the point of entrance to the turbine guides.

As will be shown later, when the theory is presented, the use of

out any loss of head.

Since the wheel passages of an impulse turbine must be open to the air it is readily seen that the use of a draft tube in the usual sense of the word is not possible. However, as will be seen later, the impulse turbine is better suited for comparatively high heads so that the loss from the wheel to the tail water is a relatively unimportant item.

13. Flumes and Penstocks.—If the turbine be used under a head of about 30 ft. or less a flume may conduct the water to an open pit as in Fig. 4 and Fig. 8. If the head is much greater than this it becomes uneconomical and a penstock is used as in Fig. 10. The turbine must then be enclosed in a water-tight case. Various forms of cases will be described in Chapter V.

For penstocks where the pressure head is less than about 230 ft. (100 lb. per square inch) wood-stave pipe is frequently used. It is cheaper than metal pipe for similar service.

Cast-iron pipe is used for heads up to about 400 ft. It is not good in large diameters nor for high pressures on account of porosity, defects in casting, and low tensile strength. Its advantages are durability and the possibility of readily obtaining odd shapes if such are desired.

For high heads, steel pipe, either riveted or welded, is used. It is cheaper than cast iron in large sizes but it corrodes more rapidly.

14. QUESTIONS

1. In what ways may turbines be classified? How many of these are found in current practice? Explain the features of each.

2. What are the differences between impulse and reaction turbines? What types of each are now used? Explain the various directions of flow that may be used.

3. What are the relative merits of horizontal and vertical shaft turbines? When would each ordinarily be used?

4. What arrangements of runners may we have for vertical shaft units? For horizontal shaft units? What is meant by single- and double-overhung construction?

5. What two functions does the draft tube fulfill? How does it prevent

15. Investigation.—Before a water-power plant is erected a
reful study should be made of the stream to determine the
rse-power that may be safely developed. It is important to
now not only the average flow but also both extremes. The
treme low-water stage and its duration will determine the
nount of storage or auxiliary power that may be necessary.
ie extreme high-water stage will fix the spillway capacities of
ms, determine necessary elevations of machines, and other
cts essential to the safety and continuous operation of the plant.

16. Rating Curve.—The first step in such an investigation is
e establishment of a rating curve. (See Fig. 12.) To determine

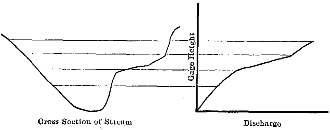


Fig. 12.—Rating curve.

e discharge of the stream a weir, current meter, floats, or other
eans may be employed according to circumstances.[1]

By measuring the flow of the stream for different stages a
ting curve is readily drawn. This will not be a smooth curve
there are abrupt changes in the area of the section. A given
ge height may really represent a range of flows depending upon
hether the river is rising or falling, the flow being greater if the
eam is rising and less if it is falling. This is because the
draulic gradient is different in the two cases.[2] If possible,

Hoyt and Grover, "River Discharge."

Water Supply Papers No. 94 and No. 95 of the U. S. G. S.

If the bed of the stream changes, as it frequently does in sandy or alluvial soil, the rating curve will also change and must be determined anew from time to time. Sometimes a special permanent control station may be constructed to avoid this.

17. The Hydrograph.—When gage readings are taken regularly and frequently for any length of time and the corresponding

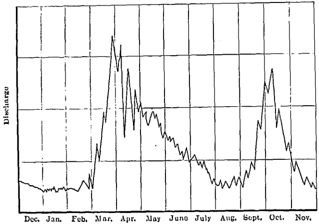


Fig. 13.—Hydrograph

discharges secured from the rating curve a history of the flow may be plotted as in Fig. 13. Such a curve is called a hydrograph. This curve is extremely useful in the study of a water-power proposition. To be satisfactory it should cover a period of several years since the flow will vary from year to year. Since it is very important to know the extremes also, it should cover both a very dry year and a very wet one as well as the more normal periods.

18. Rainfall and Run-off.—Rainfall records are usually available for many years back and are a valuable aid in extending the scope of the hydrograph taken, provided a relation between rainfall and run-off can be estimated. If the ground be frozen,

ually the conditions are such that the relation is between these
o extremes. In a general way it may be said to lie between the
o curves shown in Fig. 14.[1]

The relation between rainfall and run-off is very complicated
d only partially understood at present. For more information
nsult Water Supply Papers of the U. S. G. S. and other sources.

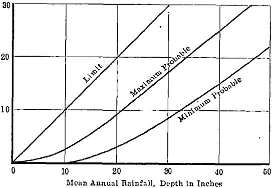


FIG. 14.—Relation of rainfall to run-off.

29. Absence of Satisfactory Hydrograph.—If no hydrograph
the stream is available and there is no time to secure one, a
dy of the stream may be made by comparison with the hydro-
phs of adjacent streams. It is well, however, to take a
drograph for a year, if possible, in order to be able to check
comparison.

f no hydrographs of adjacent streams are available, it is neces-
y to use the rainfall records and make a thorough study of
physical conditions of the water shed. If the relation between
nfall and run-off can be estimated, then fairly satisfactory
clusions may be drawn, provided a hydrograph for one year
be used to work from. Where there is not time to take a
r's record it is well to be very conservative and provide for
ure extension of power if it is later found to be warranted.

0. Variation of Head.—Since the discharge of any stream is

of high water, the head water level rose the same amount as the tail water level the net head under which the turbine operated would remain constant. But, under the usual conditions, the tail water level rises more than the head water level and the net head under which the turbine operates becomes less. This is illustrated in Fig. 15 where three rates of flow are shown.

At high water the horse-power of the stream may be large even though the fall be reduced, owing to the increased quantity of

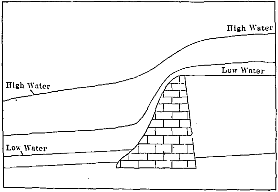


FIG. 15.—Decrease of available head at high water.

water. But the horse-power of the turbine may be seriously diminished. A turbine is only a special form of orifice and therefore the discharge through it is proportional to the square root of the head. If then the discharge through it be reduced due to the lower head, the horse-power input to the turbine is decreased. If the best efficiency is to be obtained, the speed also should vary as the square root of the head. But usually the turbine is compelled to run at constant speed and this causes a further reduction of the power of the turbine since the efficiency is lowered. (The speed should be the best for low water because economy of water is then important.) It is thus seen that the decrease of the head at high water causes a loss of power and a drop in efficiency. This

mputed from the hydrograph by using the heads that would be

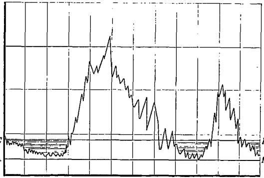


FIG. 16.—Power curve of a stream.

tained at various stages of flow. Or the hydrograph itself may ll be used as a power curve if the power scale that is used is ade to vary as the head varies instead of being uniform.

If Fig. 16 represents the power curve of a stream then A–B oresents the greatest power that the stream can be counted on to furnish at all times.

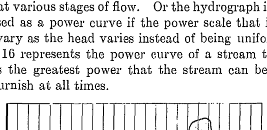


FIG. 17.

22. Pondage and Load Curve.—By pondage is meant the ring of a limited amount of water. If the plant be operated

while the peak load may be much greater.

23. Storage.—By storage is meant the storing of a considerable quantity of water, so that it varies from pondage in degree only. Pondage indicates merely sufficient capacity to supply water for a few hours or perhaps a few days, but storage implies a capacity which can supply water needed during a dry spell of several weeks or months or more. The effect of storage is to enable the minimum power of the stream to be raised from A–B to C–D (Fig. 16). The greater the storage capacity the higher C–D is placed until it equals the average power of the stream. The water for the turbines may be drawn direct from the storage reservoir (in which case the head varies) or the reservoir may be used as a stream feeder only.

A plant operating under a low head requires a relatively large amount of water for a given amount of power. A storage basin for such a plant would require a very large capacity if it were to furnish power for any length of time. But a low head is usually found in a fairly flat country where it is possible to construct a storage reservoir of limited capacity only, and often none at all, on account of flooding the surrounding country. But for a high head the conditions are different as only a relatively small amount of water is required so that the capacity of the storage reservoir need not be excessive. The higher the head, the more valuable a cubic foot of water becomes. The topography of a country where a high head can be developed is usually such that storage reservoirs of large capacity can be constructed at reasonable cost. A low-head plant usually possesses pondage only—a high-head plant usually possesses storage.

24. Storage and Turbine Selection.—If a plant possesses neither storage nor pondage, or the stream flow may not be interrupted because of other water rights, the economy of water when the turbine is running under part load is of no importance. The efficiency at full load is all that is of interest. But if the plant does have pondage or storage in any degree the economy of water

head at C or CX which is equal to the static head. The hydraulic

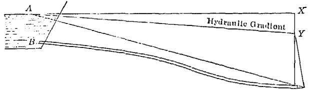


FIG. 18.—Varying rates of flow in pipe line.

gradient is then a horizontal line. If the nozzle be partially opened, so that flow takes place, the losses in the pipe line as well as the velocity head in the pipe cause the pressure to drop to CY. A further opening of the nozzle would cause the pressure to drop to a lower value. If the nozzle were removed the pres-

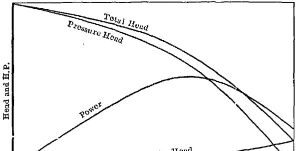


FIG. 19.—Head and power at end of pipe line.

sure at C is then atmospheric only, which we ordinarily call zero pressure. The hydraulic gradient is then A–C.

Head is the amount of energy per pound of water. The head at C is the elevation head, taken as zero, plus the pressure head, plus the velocity head. When the discharge is zero the head is a maximum, being equal to CX. When the nozzle is removed the

between these two extremes as is shown in Fig. 19. Let the rate of discharge through the pipe be denoted by q, the net head at C by h, the loss of head by H', and the height CX by z. If the loss of head in the pipe be assumed proportional to the square of the velocity of flow we may write $H' = Kq^2$, where K is a constant whose value depends upon the length, size, and nature of the pipe. Then

$$\text{Power} = qh = q(z - H') = qz - Kq^3$$

Differentiating $\quad d(\text{Power})/dq = z - 3Kq^2 = 0$

Or $\quad z = 3Kq^2 = 3H'.$

Thus the power delivered by a given pipe line is a maximum when the flow of water is such that one-third the head available is used up in pipe friction, leaving the net head only two-thirds of that available.

The efficiency of the pipe line is expressed by h/z. Thus in the case where the pipe line is delivering its maximum power, its efficiency is only 66⅔ per cent. But if economy in the use of water is an object the discharge through the pipe would be kept at a lower value than this so as to prevent so much of the energy of the water being wasted. For a given quantity of water, this means that a larger pipe would be used, so that its efficiency would be higher. In a similar manner, if a given amount of power is required, the smallest pipe that can be used will be of such a size that its efficiency is 66⅔ per cent. As the pipe is made larger than this, its efficiency rises and the amount of water required decreases.[1]

The most economical size of pipe may be found as shown in Fig. 20. One curve represents the annual value of the power lost

[1] It should be noted that in this paragraph there are three separate cases mentioned. First the size of the pipe is fixed and different rates of discharge are assumed to flow through it. Second the quantity of water available is fixed and the size of the pipe is the variable. Third the power delivered is

which this sum is a minimum is the most economical.

If the rate of discharge is not constant, careful study must be made of the load curve in order to determine what value of e rate of discharge will give the average power lost. For the pical load curve this value may often be found to be about 80 r cent. of the maximum flow.

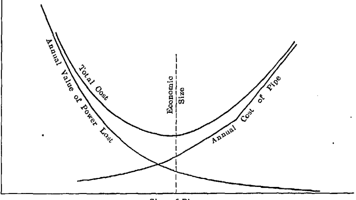


FIG. 20.—Determination of economic size of pipe.

It must be noted that this solution may not always be the most ctical because of other considerations. For instance the ocity of the water may be too high and thus give rise to uble due to water hammer. Again if the loss of head is too ge a percentage of the head available, the variation of the net ad between full discharge and no discharge may be conerable. This might cause trouble in governing the turbine.

26. Pipe Line and Speed Regulation.—A fundamental propoon in mechanics is that

$$\text{input} = \text{output} + \text{losses} + \text{gain in energy}.$$

he speed of a turbine is to remain constant it follows that the

If the load on a turbine is rapidly reduced the quantity of water supplied to it must be very quickly decreased in order to keep the speed variation small. This means that the momentum of the entire mass of water in the penstock and draft tube must be suddenly diminished. If the penstock be long a big rise in pressure may be produced so that momentarily the pressure may be greater than the static pressure. This increase in pressure may be sufficient to even cause an increase in the power input for a very brief interval of time. On the other hand, if the load on the turbine be suddenly increased, the water in the penstock and draft tube must be accelerated and this causes a temporary drop in pressure below the normal value, and for the time being the power input to the turbine may be diminished below its former value. The longer the pipe line and the higher the maximum velocity of flow, the worse these effects become. It is thus seen that the speed regulation depends upon the penstock and draft tube as well as upon the governor and the turbine.[1]

If the velocity of the water is checked too suddenly a dangerous water hammer may be produced. In order to avoid an excessive rise in pressure, relief valves are often provided. Automatic relief valves are analogous to safety valves on boilers; they do not open until a certain pressure has been attained. Mechanically operated relief valves are opened by the governor at the same time the turbine gates are closed and afford the water a by-pass so that there is no sudden reduction of flow. To prevent waste of water these by-passes may be slowly closed by some auxiliary device. Another means of equalizing these pressure variations is to place near the turbine a stand pipe or a surge chamber, with compressed air in its upper portion, or open to the atmosphere if it can be made high enough. These have the advantage over the relief valves that they are not only able to prevent the pressure in-

[1] A case may be cited where the length of a conduit was 7.76 miles, the average cross-section 100 sq. ft., and the maximum velocity 10 ft. per second. The amount of water in the conduit was, therefore, 128,125 tons and with

1. Before a water power plant is built what information should be obtained regarding the stream? How may this be determined?

2. What is the rating curve of a stream? How is it obtained? What use is made of it? Is it always the same?

3. What is the hydrograph? How is it obtained? What is its use?

4. What use may be made of rainfall records, if a hydrograph of the stream has been obtained by direct measurement? What use may be made of rainfall records, if no hydrograph is in existence?

5. Is the head on a water power plant constant? What causes this? Do the head water levels and the tail water levels change at the same rate? Why? What effect does this have on the power and efficiency of the turbine? What types of plants are most seriously affected?

6. How is the power of a stream to be determined? What effect does pondage have upon this? What is the difference between pondage and storage and how do they differ in their effects upon the extent of the power development?

7. As the flow of water through a given pipe increases, how do the head and power delivered change? How does the efficiency vary? For what condition is the power a maximum? Is this desirable?

8. If a given rate of discharge is to be used for power, how may the proper size of pipe be determined? Are there several factors that need to be considered?

9. If a given amount of power is required and the water supply is ample, how can the smallest size of pipe that would serve be found? What would limit the largest size that might be used?

10. How does the head on a turbine change with the load the wheel carries? What effect does the pipe line have upon speed regulation?

11. What devices are employed to care for the condition when the governor suddenly diminishes the water supply? What may be used to care for a sudden demand?

12. The following table gives the results of a current meter traverse of a stream: Velocity of water in ft. per second equals 2.2 times revolutions per second of the meter plus 0.03.

From this data compute the area, rate of discharge, and mean velocity of the stream. (The mean velocity in a vertical ordinate will be found at about 0.6 the depth. The mean velocity is obtained with a slightly greater degree of accuracy by taking the mean of readings at 0.2 and 0.8 the depth.

[1] See "Control of Surges in Water Conduits," by W. F. Durand, *Journal A. S. M. E.*, June, 1911; "The Differential Surge Tank," by R. D. Johnson, *Trans. A. S. C. E.*, Vol. 78, p. 760, 1915; and "Pressure in Penstocks

20	0.9	0.54	48	20	80	1.3	0.26	51	20
25	1.5	0.30	48	20			1.04	44	10
		1.20	42	15	85	1.4	0.28	52	20
30	1.7	0.34	41	30			1.12	43	10
		1.36	48	30	90	1.2	0.24	49	20
35	1.9	0.38	45	30			0.96	53	15
		1.52	50	20	95	1.3	0.26	40	15
40	1.8	0.36	45	30			1.04	30	10
		1.44	43	20	100	1.1	0.22	45	20
45	1.7	0.34	49	30			0.88	56	15
		1.36	45	20	105	1.0	0.20	45	20
50	1.6	0.32	42	30			0.80	55	15
		1.28	43	20	110	1.2	0.24	46	20
55	1.5	0.30	50	30			0.96	50	10
		1.20	49	20	115	1.2	0.24	41	15
60	1.6	0.32	53	30			0.96	58	10
		1.28	52	15	120	0.8	0.48	55	5
65	1.4	0.28	55	30	125	0.9	0.54	47	5
......	......	1.12	55	15	130	1.1	0.66	42	2
......	......	......	......	......	135	1.1	......	......	......
......	......	......	......	......	140	0.0	......	......	......

The area between two ordinates may be taken as the product of the distance between them by half the sum of the two depths. The mean velocity in such an area may be taken as half the sum of the mean velocities of the ordinates. The product of area and mean velocity gives the discharge through the area. The sum of all such partial areas and discharges gives the total area and total discharge of the stream. The total discharge divided by the total area gives the mean velocity of the stream.)

Ans. 171.8 sq. ft., 146.8 cu. ft. per second.

13. The traverse of the stream given in problem (12) was made May 14, 1913 when the gage height was 1.21 ft. Other ratings had been made as noted.

Date	Width, ft.	Area, sq. ft.	Mean velocity, ft. per sec.	Gage height, ft.	Discharge, sec.-ft.
November 3, 1906..........	138	485	......	3.10	1345
May 10, 1908..............	138	345	......	2.32	758
September 4, 1908..........	60	45	......	0.57	24
July 24, 1909..............	138	157	......	1.12	150
November 19, 1909.........	72	60	......	0.74	51
May 12, 1910..............	138	226	......	1.63	364
October 11, 1911...........	138	165	......	1.32	202
July 30, 1912..............	78	49	......	0.70	42

bed of the stream. From the data given draw to scale the probable out- of the cross-section of the stream. Plot values of area, mean velocity, discharge aga'nst gage height. (The area and velocity curves can be nded with greater assurance than the discharge curve. By computing es of discharge from these two, the discharge curve may be produced ond readings taken.)

4. The daily gage heights of the stream of the preceding problem for 2 are given below. Plot the hydrograph. Note values of maximum, mum, and average flow, and the duration of the minimum flow.

Day	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1	1.10	1.48	6.50	2.30	2.70	1.32	0.80	0.80	1.04	1.22	1.48	1.31
2	1.20	1.46	5.70	2.00	2.30	1.25	0.85	0.74	1.25	1.24	1.60	1.34
3	1.10	1.48	4.00	3.80	2.20	2.30	0.82	0.76	1.34	1.16	1.48	2.30
4	1.30	1.38	2.00	2.90	2.16	.....	0.90	0.79	1.14	1.12	1.42	1.90
5	1.05	1.35	1.80	2.60	1.90	1.55	0.86	0.74	1.06	1.14	1.31	1.70
6	1.10	1.32	1.55	2.40	1.85	1.50	0.84	0.71	1.01	1.00	1.28	2.10
7	2.20	1.30	1.15	2.35	1.95	1.80	0.80	0.71	0.92	1.01	1.31	2.10
8	2.55	1.52	1.04	2.90	2.15	1.55	0.76	0.71	0.02	0.95	2.00	1.60
9	2.30	1.58	1.90	2.45	2.40	1.38	0.74	0.74	0.05	1.14	1.70	1.70
10	2.50	1.58	1.95	2.25	2.20	1.29	0.72	0.79	0.91	1.05	1.55	1.65
11	1.85	1.44	1.70	2.15	1.95	1.24	0.74	2.35	0.92	1.00	1.50	1.55
12	2.00	1.48	1.70	2.10	1.80	1.21	0.74	1.50	0.90	1.10	1.41	1.49
13	1.85	1.52	3.60	2.05	1.80	1.21	0.72	1.05	0.90	1.11	1.42	1.35
14	1.85	1.60	3.10	2.00	1.70	1.15	0.88	1.20	0.85	1.08	1.50	1.48
15	2.00	1.50	3.20	2.05	2.00	1.12	0.88	1.00	0.85	1.01	1.46	1.35
16	1.80	1.52	4.50	.. ..	2.30	1.15	0.83	0.95	0.88	0.05	1.30	1.40
17	1.75	1.55	3.20	2.35	2.00	1.00	0.80	1.42	0.86	0.89	1.38	1.38
18	1.85	1.58	3.00	3.00	1.80	1.11	1.00	1.15	0.94	0.01	1.04	1.34
19	2.20	1.56	2.80	3.06	1.70	1.05	0.08	1.42	1.15	1.01	1.28	2.25
20	2.80	1.55	2.60	2.70	1.05	1.02	0.84	1.30	1.14	0.96	1.38	2.10
21	2.86	1.58	2.25	2.35	1.60	1.00	0.85	1.12	1.01	0.96	1.28	1.90
22	6.30	2.42	2.30	2.10	1.60	1.02	1.02	1.20	1.04	0.92	1.26	1.75
23	3.35	2.20	1.95	2.25	1.48	0.94	0.95	1.06	1.01	1.35	1.22	1.70
24	2.55	2.30	2.05	2.10	1.43	0.05	0.76	1.14	1.18	2.80	1.36	1.60
25	2.16	2.22	2.22	2.10	1.45	0.95	0.84	1.04	2.05	2.60	1.70	1.55
26	1.95	2.20	1.95	1.90	1.70	0.94	0.84	0.99	1.70	2.25	1.60	1.60
27	1.95	2.05	2.00	2.00	1.55	0.98	0.81	0.96	1.45	1.95	1.50	1.70
28	1.80	2.00	2.20	1 00	1.40	0.88	0.80	1.04	1.32	1.80	1.42	1.80
29	1 68	2 00	4.20	1.80	1.42	0.90	0.70	1.00	1.28	1.70	1.40	1.65
30	1.60	.....	3.60	3.20	1.05	0.89	0.75	0.91	1.26	1.60	1.35	1.70
31	1.62	.....	3.00	.....	1.38	.....	0.72	0.99	.....	1.50	.....	2.50

5. The following table gives the rainfall record in a certain vicinity for

	Rainfa inche	Run-o inche	Rainfa inche	Run-o inche	Rainfa inche	Run-o inche	Rainfa inche	Run-o inche	Rainfa inche	Run-o inche	Rainf inch	Run-o inch
Jan	3.05	1.0	3.21	2.0	4.14	2.2	1.15	1.0	2.85	1.8	4.01	3.5
Feb.......	1.95	1.8	4.61	3.5	5.17	4.0	1.84	1.7	2.11	1.8	4.04	3.0
March....	1.91	2.6	4.04	4.0	3.74	3.6	1.48	2.3	2.98	3.2	5.16	4.7
April......	2.19	2.2	3.78	3.1	4.91	3.6	5.96	4.0	2.82	2.6	5.71	3.9
May......	2.72	1.2	4.98	2.0	2.94	1.3	2.58	1.2	1.33	0.0	3.15	1.3
June......	2.73	0.8	1.53	0.6	3.50	0.9	3.47	0.9	7.98	2.3	1.32	0.6
July......	2.74	0.4	3.44	0.4	1.86	0.3	2.00	0.3	3.03	0.4	3.14	0.4
Aug.......	2.55	0.4	2.66	0.4	3.66	0.5	2.80	0.4	5.70	0.9	6.30	1.0
Sept......	6.88	1.7	4.04	0.8	2.73	0.5	3.38	0.6	3.57	0.7	4.40	0.9
Oct.......	4.69	1.7	1.40	0.5	1.28	0.4	1.20	0.4	5.33	1.2	3.56	0.8
Nov.......	4.70	1.6	2.51	0.8	1.75	0.6	3.15	1.0	3.06	1.0	2.32	0.8
Dec.......	4.88	2.5	0.00	0.5	2.93	1.4	1.03	1.1	3.20	1.6	4.02	2.0

16. The present capacity of the Lake Spaulding reservoir of the Pacific Gas and Electric Co. is 2,000,000,000 cu. ft. (it will eventually be twice this), the present flow is 300 cu. ft. per second, and the net head on the power house is approximately 1300 ft. If the plant runs at full load continuously and there is no stream flow into the lake, how long would this water last? If this same storage capacity were available for a plant of the same power under a head of 40 ft., what rate of discharge would be required and how long would the water last? (It is worth noting that the surface area of Lake Spaulding is 1.3 square miles and the total drop in the water surface would be 56 feet if the sides were vertical. Actually the drop is greater. No such drop in level would be found in connection with a plant under a 40-ft. head. If we assume the drop in level to be 10 ft., for example, the surface area of the storage reservoir would have to be 233 square miles. Also the lowering of the head on the plant in the latter case would make it necessary to use more water and hence shorten the time as computed.)

Ans. 772 days, 9750 cu. ft. per second, 2.4 days.

17. The difference in elevation between the surface of the water in a storage reservoir and the intake to a turbine was 132.4 ft. During a test the pressure at the latter point was 126.6 ft. and the discharge 44.5 cu. ft. per second, giving a velocity head in a 30 in. intake of 1.3 ft. What was the efficiency of the pipe line? What was the value of the power delivered?

Ans. 96.6 per cent., 647 h.p.

18. Assuming the loss of head to be proportional to the square of the rate of discharge, what is the maximum power the pipe in problem (17) could deliver? How many cubic feet of water per second are consumed per horsepower in problems (17) and (18)? *Ans.* 1400 h.p., 0.069, 0.088.

second in every case. Assume this expression to be true for similar pipes of different sizes, the cost of 3, 4, 5, and 6 ft. riveted steel pipes to be \$4.25, \$7.50, \$12.50, and \$18.00 per foot respectively, and the length of pipe to be 2000 ft. If the value of a horsepower per year is \$20, the interest and depreciation rate 7 per cent., and the rate of discharge 44.5 cu. ft. per second, what is the most economical size of pipe? *Ans.* 5 ft.

THE TANGENTIAL WATER WHEEL

28. Development.—The tangential water wheel is the type of impulse turbine used in this country. Its theory and characteristics are precisely the same as those for the Girard impulse turbine, used abroad, and the two differ only in appearance and mechanical construction. It is used rather than the Girard turbine, because of the advantages offered by its superior type

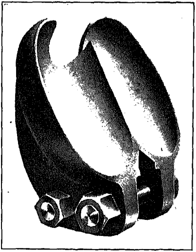


FIG. 21.—Doble ellipsoidal bucket.

of construction. The tangential wheel is also called an impulse wheel or a Pelton wheel in honor of the man who contributed to its early development. The use of the term "Pelton water wheel" does not necessarily imply, therefore, that it is the product of the particular company of that name.

The development of this wheel was begun in the early days in California but the present wheel is a product of the last 20 years.

right in the center. A man by the name of Pelton was running one of these wheels one day when it came loose on its shaft and slipped over so that the water struck it on one edge and was dis-

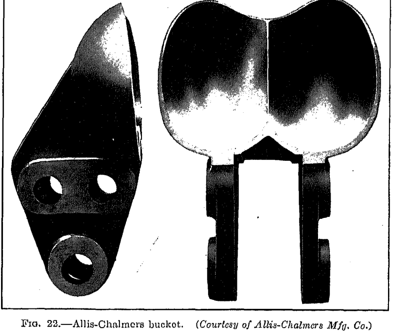


FIG. 22.—Allis-Chalmers bucket. (*Courtesy of Allis-Chalmers Mfg. Co.*)

charged from the other edge. The wheel was observed to pick up in power and speed and this led to the development of the split bucket.

29. Buckets.—The original type of Pelton bucket may be seen in Fig. 76, page 88, the Doble ellipsoidal bucket is shown in Fig. 21, the Allis-Chalmers type in Fig. 22, while the recent Pelton bucket may be seen in Fig. 23. In every case the jet strikes the dividing ridge and is split into two halves. The

serves two buckets in such a fashion that the latter are connected

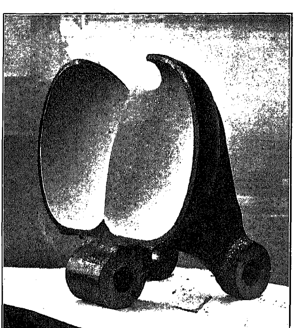


Fig. 23.—Pelton bucket. (*Courtesy of Pelton Water Wheel Co.*)

as a chain. The advantage gained is one of compactness, it being possible to place the buckets somewhat closer together.

30. General Proportions.—It has been found that for the best efficiency the area of the jet should not exceed 0.1 the projected area of the bucket, or the diameter of the jet should not exceed 0.3 the width of the bucket.[1] If this ratio is exceeded the buckets are crowded and the hydraulic friction loss becomes excessive. It is evident also that there must be some relation between size of jet and the size of the wheel. For a given size jet there is no

In special cases, where a low r.p.m. was desired, diameters as large as 35 ft. have been used when the diameter of the jet was only a few inches. But there is a lower limit for the ratio of wheel diameter to jet diameter. Obviously, for instance, the wheel could not be as small as the jet. The considerations which influence this matter will be further considered in Chapter VII, but for the present it will be sufficient to state that a ratio as low as 9 may be used without an excessive loss of efficiency.[1] (The nominal diameter is that of a circle tangent to the center

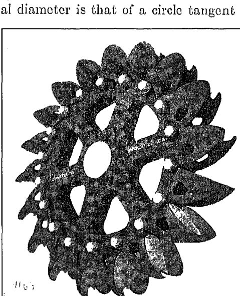


FIG. 24.—Pelton tangential water wheel runner showing interlocking chain-type construction. (*Made by Pelton Water Wheel Co.*)

line of the jet.) The more common value, and one which involves no sacrifice of efficiency, is 12. From that we get a very convenient rule that the diameter of the wheel in feet equals the diameter of the jet in inches. The size of jet necessary to

wheel or to use a larger wheel with a single jet. The larger wheel

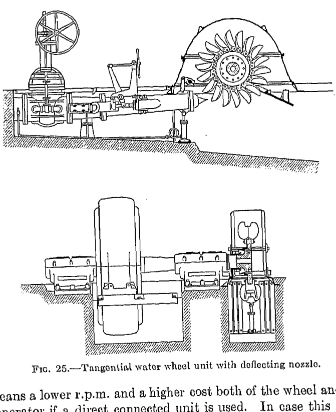
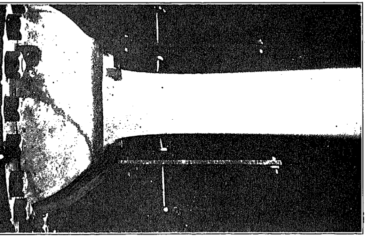


FIG. 25.—Tangential water wheel unit with deflecting nozzle.

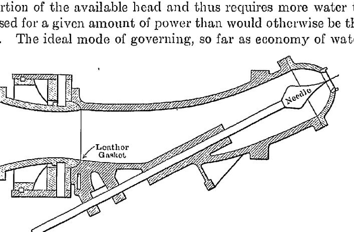
means a lower r.p.m. and a higher cost both of the wheel and the generator if a direct connected unit is used. In case this additional expense is not justified by the increased efficiency of the single jet wheel the duplex nozzle would be used.

The tangential water wheel is almost always set with a horizontal shaft and, if direct connected to a generator, is overhung so that the unit has only two bearings (Fig. 25). It is quite common for two wheels to drive a single generator mounted between them in which case we have the double-overhung type.



26.—5286 h.p. Jet, from $7\frac{1}{2}$ in. needle nozzle. Head = 822 ft. Jet velocity = 227.4 ft. per second.

of any throttle valve in the pipe line is wasteful as it destroys rtion of the available head and thus requires more water to sed for a given amount of power than would otherwise be the . The ideal mode of governing, so far as economy of water



27.—Deflecting needle nozzle. (*After drawing by Prof. W. R. Eckart, Jr.*)

nearly the same for all values of discharge. The efficiency of a well-constructed needle nozzle is very high, being from 95 to 98 per cent.[1] The needle nozzle is nearly ideal for economy of water but may not always permit close speed regulation. If the pipe line is not too long, the velocity of flow low, and the changes of load small and gradual, the needle nozzle may be very satisfactory. In case it is used the penstock is usually provided with a standpipe or a surge tank.

If the pipe line is long, the velocity of flow high, and the changes of load severe, dangerous water hammer might be set up if the discharge were changed too quickly. It might therefore be difficult to secure close speed regulation with the needle nozzle as the governors would have to act slowly. The deflecting nozzle, shown in Fig. 7, page 8, is much used for such cases. The nozzle is made with a ball-and-socket joint so that the entire jet can be deflected below the wheel if necessary. The governor sets the nozzle in such a position that just enough water strikes the buckets to supply the power demanded. The rest of the water passes below the buckets and is wasted. Since there is no change in the flow in the pipe line the governor may accomplish any degree of speed regulation desired as there is little limit to the rapidity with which the jet may be deflected. Such a nozzle is usually provided with a needle also which is regulated by hand. Fig. 27 is really of this type. In another type the body of the nozzle is stationary and only the tip is moved. The needle stem must be equipped with a guide in this moving part and also be fitted with a universal joint so that the needle point may always remain in the center of the jet. The station attendant sets the needle from time to time according to the load that he expects to carry. However, the device is wasteful of water unless carefully watched. If other water rights prevent the flow of a stream from being interfered with it may be satisfactory.

In some modern plants the operator can control the position of the needle from the switchboard and by careful attention very

The combined needle and deflecting nozzle may possess the advantages of both of the above types, by having the needle automatically operated. If the load on the wheel is reduced

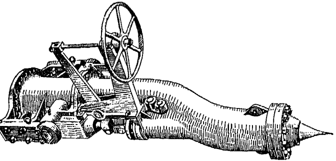


FIG. 28.—Deflecting needle nozzle for 8000 h.p. wheel.

the governor at once deflects the jet thus preventing any increase of speed. Then a secondary relay device slowly closes the needle nozzle and, as it does so, the nozzle is gradually brought back to its original position where all the water is used upon the wheel. Thus close speed regulation is accomplished with very little waste of water.

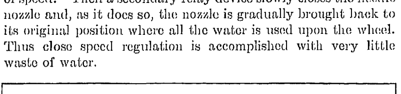


FIG. 29.—Needles and nozzle tips. (*Courtesy of Pelton Water Wheel Co.*)

The needle nozzle with auxiliary relief shown in Fig. 30 and Fig. 31 accomplishes the same results as the above. When the needle

load they are unable to afford any assistance in the case of a rapid demand for water. The deflecting nozzle alone is the only type that is perfect there.

32. Conditions of Use.—The tangential water wheel is best adapted for high heads and relatively small quantities of water. By that is meant that the choice of the type of turbine is a function of the capacity as well as the head. For a given head the larger the horsepower, the less reason there is for using this type of wheel.

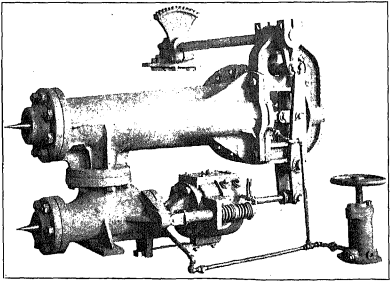


FIG. 30.—Auxiliary relief needle nozzle.
(*Made by Pelton Water Wheel Co.*)

In Switzerland a head as high as 5412 ft. has been used for 5 wheels of 3000 h.p. each. The jets are 1.5 in. in diameter and the wheels, which run at 500 r.p.m., are 11.5 ft. in diameter. There are several installations in this country under heads of about 2100 ft. There are numerous

is about the same as that of the average reaction turbine. From 75 to 85 per cent. may reasonably be expected though lower values are often obtained, due to poor design.

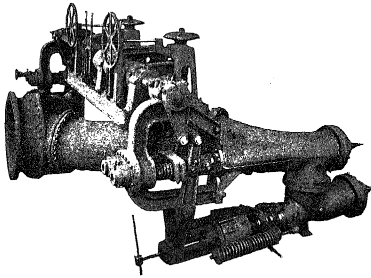


FIG. 31.—Auxiliary relief needle nozzle for use with 10,000 kw. tangential water wheel. (*Made by Pelton Water Wheel Co.*)

34. QUESTIONS AND PROBLEMS

1. Of what materials are impulse wheel buckets constructed? How are they secured to the rim? What is the advantage of "chain type" construction? What should be the relation between the size of the jet and the size of the bucket?

2. When would two or more jets be used upon a Pelton wheel? What is the relation between the diameter of the wheel and the diameter of the jet? How may the speed of rotation of a wheel of given diameter be computed, if the head is known? What fixes the diameter of the jet that is to be employed, assuming that it is not limited by any wheel size?

3. What is meant by single-overhung and double-overhung construction? What is the advantage of the latter?

7. It is desired to develop 3880 h.p. with a Pelton wheel under a head of 900 ft. Assuming the efficiency of the wheel to be 82 per cent. and the velocity coefficient of the nozzle to be 0.98, what will be the diameter of the jet? What will then be a reasonable diameter for the wheel and its probable speed of rotation? *Ans.* 6 in., 6 ft., 345 rev. per min.

8. How small could the wheel be made in the preceding problem? What would then be its speed of rotation? If a higher speed than this is desired for the same horsepower, what construction could be employed?

9. A Pelton wheel runs at a constant speed under a head of 625 ft. The cross-section area of the jet is 0.200 sq. ft. and the nozzle friction loss is to be neglected. Suppose a throttle valve in the pipe reduces the head at the base of the nozzle from 625 ft. to 400 ft. Under these conditions the efficiency of the wheel (the speed of the wheel no longer being proper for the head) is known to be 50 per cent. Find the rate of discharge, power of jet, and power output of wheel.

Ans. 32.08 cu. ft. per second, 1458 h.p., 729 h.p.

10. A Pelton wheel runs at a constant speed under a head of 625 ft. The cross-section area of the jet is 0.200 sq. ft. and the nozzle friction loss is to be neglected. Suppose the needle of the nozzle is so adjusted as to reduce the area of the jet from 0.200 to 0.0732 sq. ft. Under these conditions the efficiency of the wheel is known to be 70 per cent. Find the rate of discharge, power of jet, and power output of wheel.

Ans. 14.67 cu. ft. per second, 1041 h.p., 729 h.p.

11. Compare the water consumed per horsepower output for the wheel in the preceding two problems. Compute the overall efficiency in each case using the head of 625 ft. *Ans.* 32 per cent., 70 per cent.

THE REACTION TURBINE

35. Development.—The primitive type of reaction turbine own as Barker's Mill is shown in Fig. 32. The reaction of the s of water from the orifices causes the device to rotate. In ler to improve the conditions of flow the arms were then curved d it became known in this form as the Scotch turbine. Then ree or more arms were used in order to increase the power, and th still further demands for power more arms were added and e orifices made somewhat larger until the final result was a nplete wheel. In 1826 a French gineer, Fourneyron, placed stationy guide vanes within the center to ect the water as it flowed into the eel and we then had the outward w turbine. In 1843 the first Fouryron turbines were built in America. The axial flow turbine commonly led the Jonval was also a Europn design introduced into this intry in 1850.

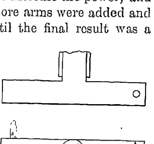


Fig. 32.—Barker's mill.

An inward flow turbine was proposed by Poncelot in 1826 but first one was actually built by Howd, of New York, in 1838. e latter obtained a patent and installed several wheels of crude rkmanship in the New England mills. In 1849 James B. ancis designed a turbine under this patent but his wheel was superior construction. Furthermore he conducted accurate ts, published the results, analyzed them, and formulated rules turbine runner design. He thus brought this type of wheel the attention of the engineering world and hence his name ame attached to it.

The original Francis turbine is shown in Fig. 5, page 4, and Fig. 6, page 5, may be seen photographs of a radial inward

runner in present practice is to be seen in Fig. 34, Type I, and in Fig. 36. The pure radial flow turbine is no longer built, but since all the modern inward mixed flow turbines may be said to have grown out of it, they are today quite generally known as Francis turbines.

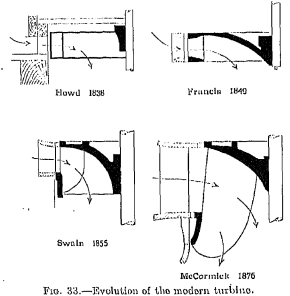
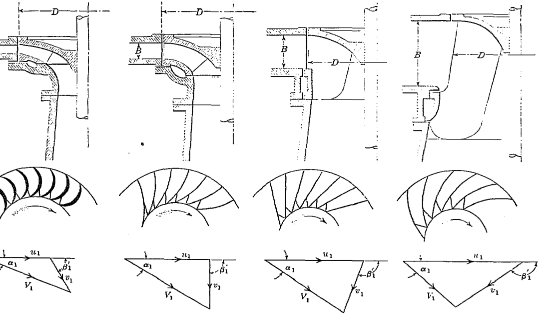


FIG. 33.—Evolution of the modern turbine.

The high-speed mixed flow runner, illustrated by the original McCormick type in Fig. 33, arose as the result of a demand for higher speed and power under the low falls first used in this country. Higher speed of rotation could be obtained by using runners of smaller diameter, but higher power required runners of larger diameter, so long as the same designs were adhered to. So in order to increase the capacity of a wheel of the same or smaller diameter, the design was altered by making the depth of the runner greater (*i.e.*, the dimension B, Fig. 34, was in-



Type I	Type II	Type III	Type IV
$\frac{B}{D} = \frac{1}{30}$	$\frac{B}{D} = \frac{1}{8}$ to $\frac{1}{4}$	$\frac{B}{D} = \frac{1}{4}$ to $\frac{1}{2}$	$\frac{B}{D} = \frac{1}{2}$
$\alpha_1 = 10°$ to $20°$	$\alpha_1 = 20°$ to $30°$	$\alpha_1 = 30°$ to $35°$	$\alpha_1 = 35°$ to $40°$
$\beta_1' = 60°$ to $90°$	$\beta_1' = 90° \pm$	$\beta_1' = 90°$ to $120°$	$\beta_1' = 120°$ to $135°$
$\phi = .57 \pm$	$\phi = .70 \pm$	$\phi = .80 \pm$	$\phi = .87 \pm$

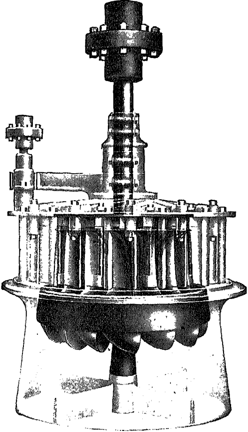


Fig. 35.—Leffel turbine for open flume. (*Made by James Leffel and Co.*)

As civilization moved from the valleys, where the low falls were found, up into the more mountainous regions, and as means of transmitting power were introduced, it became desirable to develop higher heads, and in 1890 a demand arose for high-head wheels which American builders were not able to supply.

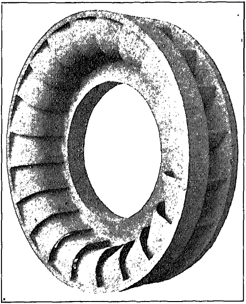
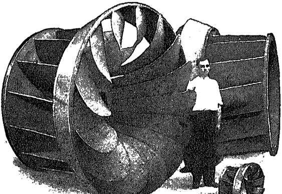


Fig. 36.—42″ Francis runner. 8000 h.p., 600 ft. head. (*Made by Platt Iron Works Co.*)



turbines largely by mathematical analysis. At the present time the best turbines in this country are designed from rational theory supplemented by experimental investigation.

36. Advantages of Inward Flow Turbine.—The Fourneyron turbine has a high efficiency on full load and is useful in some cases where a low speed is desired, but it has been supplanted by the Francis turbine for the following reasons:

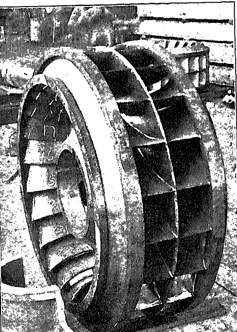
1. The inward flow turbine is much more compact, the runner can be cast in one piece, and the whole construction is better mechanically.

(*From a photograph by the author.*)
FIG. 38.—Construction of a built-up runner.

2. Since the turbine is more compact and smaller, the construction will be much cheaper. The smaller runner will permit of a higher r.p.m. and that means a cheaper generator can be used.

3. The gates for governing are more accessible and it is easier to construct them so as to minimize the losses. Thus the effi-

and the general profile as illustrated in Fig. 34. The increased volume of water through the higher capacity runners also requires a larger diameter of draft tube, as well as a higher velocity



(*From a photograph by the author.*)
Fig. 39.—Double-discharge runner.

of flow at this section, and in extreme types the flow through the runner is not merely inward and downward but for those particles of water nearest the band or ring it is inward, downward, and outward.

But the quantity of water which will flow through the runner depends not only upon the area at inlet but also upon the velocity

ϕ_e for different turbines range from about 0.55 to about 0.90 according to the design.[1] If the value of ϕ is higher than this it is probable that the speed is higher than the best speed or that the nominal diameter for which u_1 is computed is larger than the real diameter. Values of ϕ_e may be varied in the design by altering certain angles and areas of the runner.

Since it is desirable, in general, to increase or decrease the rotative speed and the capacity simultaneously, the custom is to so proportion the runners that low values of ϕ_e are found with turbines of Type I, Fig. 34, while high values are found with those of Type IV. Thus a low-capacity runner also has a low peripheral speed for a given head, while a high-capacity runner would have a higher peripheral speed. Thus for a given diameter of runner under a given head both power and speed of rotation increase from Type I to Type IV. If, on the other hand, the power is fixed, the diameter of runner of Type IV would be much smaller than that of Type I. Hence the rotative speed of the former would be higher due to the smaller diameter as well as the increased linear velocity. For this reason this type is called a high-speed runner, while Type I is a low-speed runner. Both capacity and speed are involved in a single factor variously known as the specific speed, characteristic speed, unit speed, and type characteristic. It is $N_s = N_e\sqrt{B.hp.}/h^{5/4}$, the derivation of which will be given later. (N_e is the speed for highest efficiency.) As the capacity and speed increase, this factor increases. Hence a "high-speed" turbine is really a high specific speed turbine and a "low-speed" turbine is a low specific speed turbine. The value of N_s is an index of the type of turbine. Values of N_s for reaction turbines range from 10 to 100, though the latter limit is occasionally exceeded.

The vector diagrams of the velocities at entrance are drawn to the same scale in Fig. 34 as if all four types were under the same head. It may be seen that as we proceed from Type I to Type IV, u_1, α_1, and β'_1 increase, while V_1 decreases. Since the angle

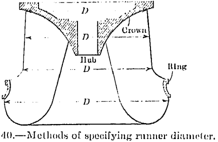


FIG. 40.—Methods of specifying runner diameter.

TABLE 1.—COMPARISON OF 12-IN. WHEELS UNDER 30-FT. HEAD

Type	Discharge, cu. ft. per minute	H.p.	R.p.m.
angential water wheel................	7.9	0.37	380
eaction turbines:			
Type I........................	99.0	4.3	460
Type II........................	329.0	14.9	554
Type III........................	741.0	33.4	600
Type IV........................	1209.0	55.5	730

TABLE 2.—COMPARISON OF WHEELS TO DEVELOP 15 H.P. UNDER 30-FT. HEAD

Type	Diameter, in.	R.p.m.
angential water wheel........................	60	55
eaction turbines:		
Type I................................	21	274
Type II................................	12	554
Type III................................	8	900
Type IV................................	6	1460

It will be seen that the tangential water wheel is a low-speed, w-capacity type, while the reaction turbine of Type IV is a high-peed, high-capacity runner. This may be contrary to the popu-

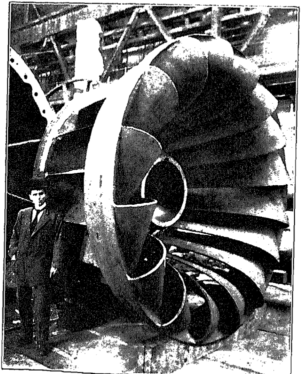


FIG. 41.—13,500 h.p. runner. Head = 53 ft., speed = 94 r.p.m. (*Made by I. P. Morris Co.*)

comes much greater. It must be understood that these tables do not prove one type of wheel to be any better than another but merely show what may be obtained. If the tangential water wheel or Type I of the reaction turbines appear in an unfavorable light it is only because the head and horsepower are not suitable for them.

majority are cast solid as the construction is more substantial. Occasionally a very large runner may be cast in sections. Built up wheels have the vanes shaped from steel plates and the crown, hubs, and rings are cast to them, as shown in Fig. 38. The best runners are made of bronze. Cast steel is used for very

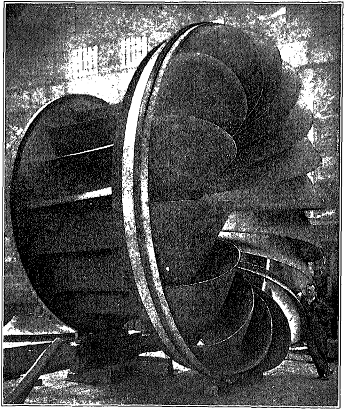


FIG. 42.—10,000 h.p. runner at Keokuk, Ia. Head = 32 ft., speed = 57.7 r.p.m. (*Made by Wellman-Seaver-Morgan Co.*)

at a point about midway down the unshaded height.

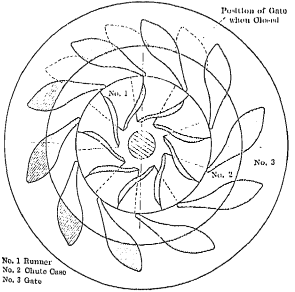
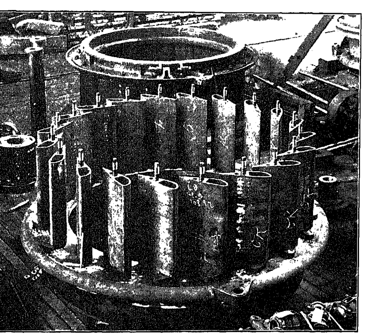


FIG. 43.—Register gate.

40. Speed Regulation.—The amount of water supplied to the reaction turbine is regulated by means of gates of which there are three types.

The cylinder gate is shown in Fig. 5, page 4. It is the simplest and cheapest form of gate and also the poorest, although, when closed, it will not leak as badly as the others. When the gate is partially closed there is a big shock loss in the water entering the turbine runner due to the sudden contraction and the sudden expansion of the stream that must take place. With this type of

ich may be rotated far enough to shut the water off entirely, necessary, as shown by the dotted lines. While this is more



(*Courtesy of Allis-Chalmers Co.*)

Fig. 44.—View of guide vanes and shifting ring.

cient than the preceding type there is still a certain amount of dy loss that cannot be avoided. It is seldom used.

The wicket gate, also called the swing gate or the pivoted ide vane, is shown in Fig. 45. This is the best type and also e most expensive. As the vanes are rotated about their pivots e area of the passages through them is altered. The vanes y be closed up so as to shut off the water if necessary. Of urse the angle, α, is altered and a certain amount of eddy loss

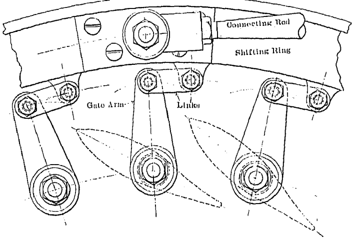
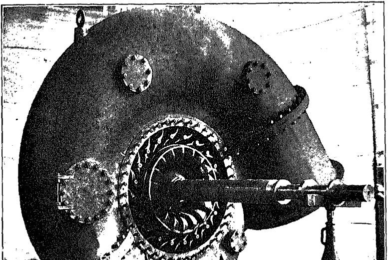


FIG. 45.—Wicket gate with all operating parts outside.



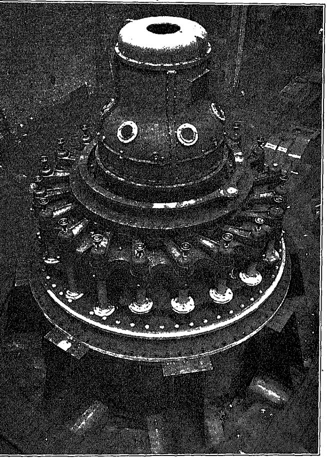


FIG. 47.—10,000 h.p. turbine at Keokuk, Ia. (*Made by Wellman-Seaver-Morgan Co.*)

are shown in Figs. 45, 47, and 48. Often the shifting ring links are inside the case, but the better, though more expensive, type has the working parts outside the case.

arranged with a dash-pot mechanism that it will slowly close.

41. Bearings.—For small vertical shaft turbines a step bearing made of lignum vitæ is used under water, as at the bottom of the runner in Fig. 35. This wood gives good results for such service and wears reasonably well. For larger turbines a thrust bearing is usually provided to which oil is supplied under pressure. Roller bearings are also used with the rollers running in an oil bath, as in Fig. 50. Sometimes rollers are provided in the former type but act only when the pressure fails, and again roller bear-

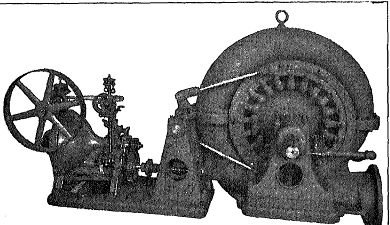
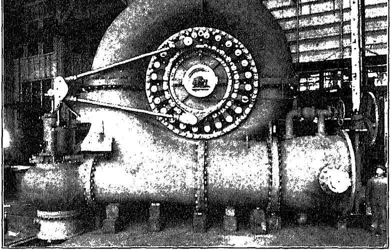
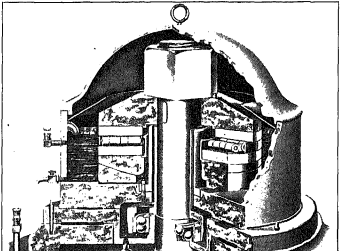


FIG. 48.—Shifting ring and links on a wicket gate spiral case turbine. (*Made by Platt Iron Works Co.*)

ings may sometimes be supplied with oil under pressure between two bearing surfaces in case the rollers fail. The Kingsbury bearing is fitted with a number of metal shoes so mounted that their bearing surfaces are not quite level. Thus as they advance through the oil bath a wedge-shaped film of oil is forced in between these shoes and the other surface. Such a bearing is preferably located at the top of the shaft in which case it is



(*Courtesy of Pelton Water Wheel Co.*)
FIG. 49.—Spiral case turbine with relief valve.



Also a single runner is often used which has a double discharge. (See Fig. 51.) Single discharge runners are often provided with some form of automatic hydraulic balancing piston to equalize the thrust.

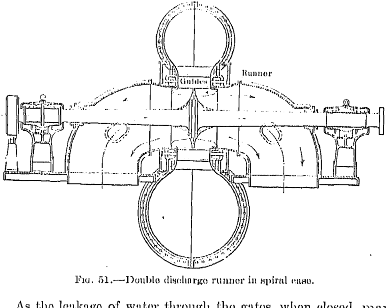


FIG. 51.—Double discharge runner in spiral case.

As the leakage of water through the gates, when closed, may be sufficient to keep the turbine running slowly under no-load, large units are often provided with brakes so they can be stopped.

42. Cases.—For low heads turbines may be used in open flumes without cases. Fig. 4, page 3, Fig. 8, page 10, and Fig. 35, page 41, are of this character. Fig. 52 shows such a type consisting of four wheels on a horizontal shaft.

Cases may also be used for very low heads and are always used for high heads. The cheapest cases are the cylinder cases (Fig. 10, page 12), and the globe cases (Fig. 53). These cases are undesirable because they permit of considerable eddy loss

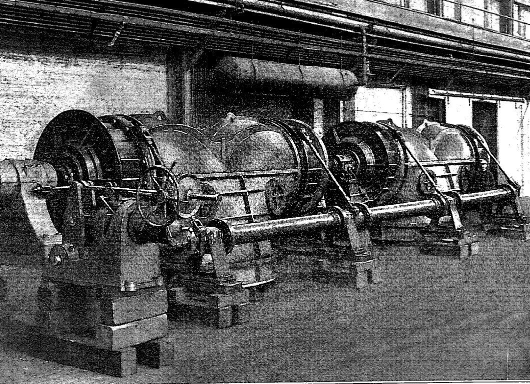


FIG. 52.—Allis-Chalmers turbine for open flume.

the guides, because only a limited portion of the water flows clear

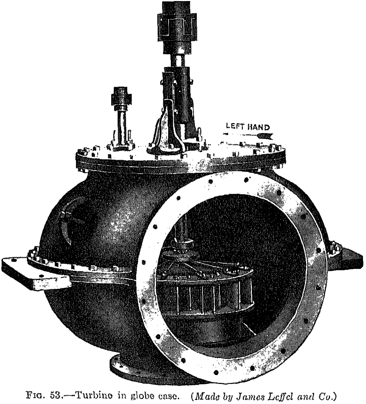


FIG. 53.—Turbine in globe case. (*Made by James Leffel and Co.*)

around to enter the further part of the circumference. Thus the average velocity throughout the case is kept the same. The case is also designed to accelerate the water somewhat as it leaves the penstock and flows to the guides.

Globe and spiral cases for low heads are made of cast iron. For higher heads they are made of cast steel as in Fig. 56. Cylinder cases (Fig. 10, page 12), are usually made of riveted sheet steel.

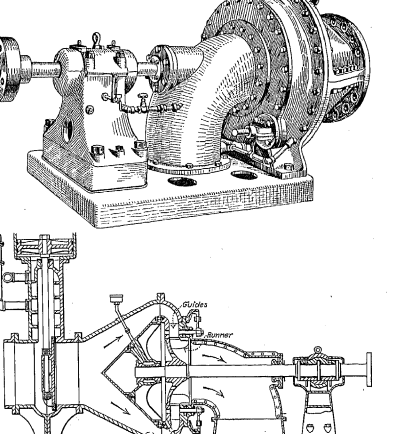


FIG. 54.—Cone case turbine.

ests upon the "speed ring." The latter consists of an upper and lower flange, as shown, which are joined together by vanes so haped as to conform to the free stream lines of the water flowing

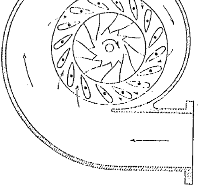
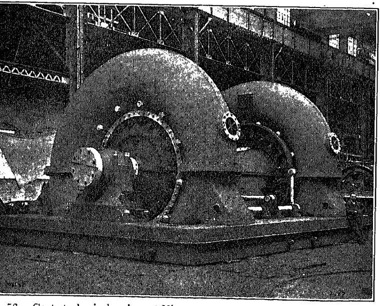
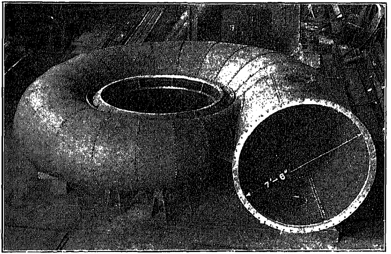


FIG. 55.—Spiral case turbine.



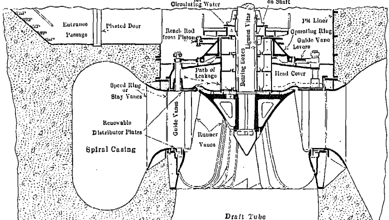
56.—Cast steel spiral casings at Niagara Falls. 14,000 h.p. at 180 ft. head. (*Made by Wellman-Seaver-Morgan Co.*)

the draft tube fulfils two distinct functions. First, it connects



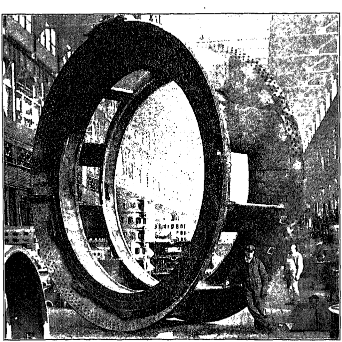
(*Courtesy of S. Morgan Smith Co*).

Fig. 57.—Spiral case of riveted steel plates.



improve the efficiency of the plant.

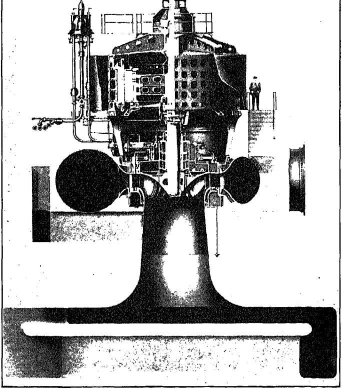
For the first purpose alone the tube might be made of a uniform cross-section, but in practice it is always made diverging so as to accomplish the second object as well. In fact, even if the runner should be set below the tail-water level, a draft tube would be of value for the second purpose. This was proven many years ago when Francis tested an outward-flow turbine with a "diffuser" surrounding the runner and found that the



(*Courtesy of Allis-Chalmers Mfg. Co.*)
FIG. 59.—Speed ring.

latter improved the efficiency by 3 per cent. As has been pointed out in Art. 37, the higher the capacity of a runner of given diameter the greater the velocity of the water must be at the

The reason for this gain in efficiency may be seen in either of the following ways. First, the total power available is that due to the fall from head-water level to tail-water level. The power of the turbine is less than this by an amount equal to that lost in the intake, penstock, and draft tube. Anything which reduces the loss outside the turbine adds just that much more to



(*Courtesy of Allis-Chalmers Mfg. Co.*)
FIG. 60.—Turbine for Niagara Falls Power Co., 37,500 h.p., 214 ft. head, 150 r.p.m.

Draft tubes are usually made of riveted steel plates as in Fig. 10, page 12, or are moulded in concrete as in Fig. 8, page 10.

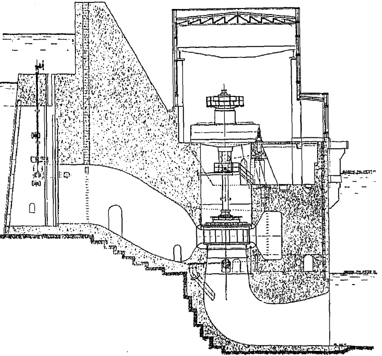
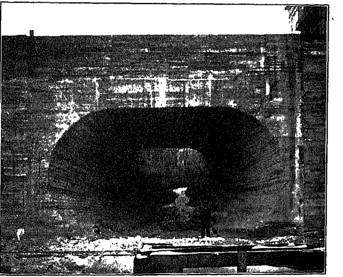


FIG. 61.—Draft tube with quarter turn.

The tube should preferably be straight but where the setting does not permit of enough room for this without excessive cost of excavation the tube is often turned so as to discharge horizontally as in Fig. 61. If the tube is large in diameter it may be necessary to make the horizontal portion of some other section than circular as in Fig. 62, in order that the vertical dimension may not be too great. A good form of section to use is oval.

d. A form that is theoretically good is "trumpet shaped," somewhat as in Fig. 60, so that the velocity of the water may be made to decrease uniformly along the length of the tube. In any event the draft tube should be so made as to secure a gradual reduction of velocity from the runner to the mouth.



(*Courtesy of Wellman-Seaver-Morgan Co.*)
FIG. 62.—Mouth of draft tube at Cedar Rapids.

The most recent innovation in draft tube construction is shown in Figs. 60 and 63. At the lower end of a comparatively short draft tube is a conoidal portion through which the water passes just before impinging on a circular plate which is concentric with the tube. The water turns and flows out along this plate around its entire circumference through an annular opening into a collecting chamber and from thence through a horizontal diverging tube to the tail race. As the water flows through the conoidal portion of the tube and impinges on the plate, its

somewhat like the spiral case, so proportioned that the water is continuously decelerated throughout the flow.

Bearing in mind that one function of the draft tube is to efficiently convert velocity head into pressure head, we see the

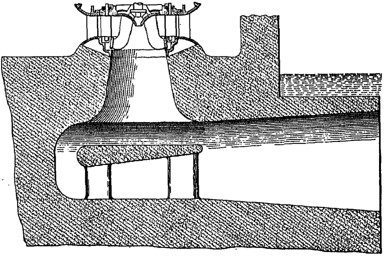


FIG. 63.—Draft tube with hydraucone.

limitations of the ordinary construction. In order to secure the diffusion desired, the length of the tube may be such that the expense of excavation is prohibitive and hence the tube is turned from vertical to horizontal with a bend of short radius. But such a bend inevitably induces eddy losses which interfere with the efficient performance of the tube. Furthermore velocity head cannot usually be converted into pressure head without a great deal of loss unless the flow of the water be smooth. Since the discharge from a turbine runner is usually quite turbulent, this alone would limit the value of a draft tube, even if it were straight. If the device just described is properly proportioned, then, as the result of hydrodynamic laws for which we have not

high-capacity type of runner is the inability of the draft tube to recover the kinetic energy of the water leaving the runner, especially in view of the fact that with this type the water leaves with some considerable "whirl." This new development may

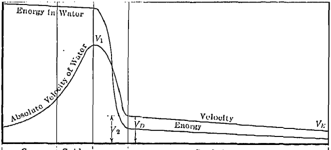


FIG. 64.—Velocity and energy transformations in turbine.

make it possible to extend the present limits of turbine runner design.

44. Velocities.—The velocities at different points are indicated by Fig. 64.[2] The velocity of flow in the penstock is determined by the consideration of the cost and other conditions in each case. The mean velocity of flow allowable in the turbine cash is as follows:

If the case is cylindrical the velocity should be as low as 0.08 to $0.12\sqrt{2gh}$ where h is the effective head. If a spiral case is used the velocity may be from 0.15 to $0.24\sqrt{2gh}$. For heads of several hundred feet the value of 0.15 is used to reduce wear on the case, 0.20 is used for moderate heads, and 0.24 is used for low heads.

The velocity at entrance to the turbine runner, $V_1 = 0.6$ to

[1] *The Journal of the Association of Engineering Societies*, vol. 27, p. 39.

[2] Mead's "Water Power Engineering."

otherwise a sudden change in velocity will take place. Velocity of discharge from the lower end of the draft tube may be about 0.10 to $0.15\sqrt{2gh}$. The value of the latter is determined by the value of the velocity at the upper end and by the length and the amount of flare to be given the tube.

45. Conditions of Use.—The reaction turbine is best adapted for a low head or a relatively large quantity of water. As was stated in Art 32, the choice of a turbine is a function of capacity as well as head. For a given head the larger the horse-power the more reason there will be for using a reaction turbine.

The use of a reaction turbine under high heads is accompanied by certain difficulties. It is necessary to build a case which is strong enough to stand the pressure; also the case, guides, and runner may be worn out in a short time by the water moving at high velocities. This depends very much upon the quality of the water. Thus a case is on record where a wheel has been operating for six years under a head of 260 ft. with clear water and the turbine is still in excellent condition. Another turbine made by the same company and according to the same design was operated under a head of 160 ft. with dirty water. In four years it was completely worn out and was replaced with an impulse wheel. The tangential water wheel has the advantage that the relative velocity of flow over its buckets is less for the same head and thus the wear is less. Also repairs can be more readily made.

The runners of reaction turbines and the buckets of impulse wheels will not last long if their design is imperfect. This is due to the fact that wherever there is an eddy or wherever there is a point of extremely low pressure, the air that is in solution in the water will always tend to be liberated at that point. And as water tends to absorb more oxygen in proportion to nitrogen than is in the air, the result is that the liberated mixture is rich in oxygen and hence readily attacks and pits the metal. Fig. 6, page 5, shows a turbine runner that has had holes eaten in it because of this reason. Thus a defective design not only produces a runner of lower efficiency because of the eddy losses

eration of reaction turbines at part gate for long periods of
ne must inevitably shorten the life of the runner.

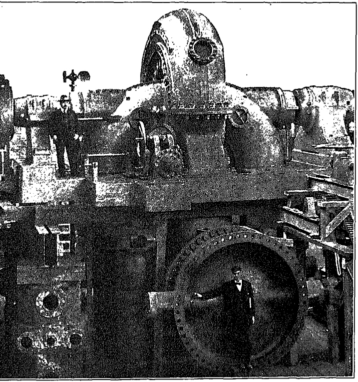


FIG. 65.—22,500 h.p. turbine for Pacific Coast Power Co. (*Made by Allis-Chalmers Mfg. Co.*)

Reaction turbines are used under very low heads in some tances. The lowest head on record is 16 in. but several feet the usual minimum. The highest head yet employed for a ction turbine is 800 ft. The latter is used for two 22,500

and is for the Niagara Falls Power Co.

There are at present quite a number of turbines in operation whose power ranges from 20,000 to 30,000 h.p.

The power of a turbine depends not only upon its size but also upon the head under which it operates. The turbines above are the most powerful, but they are not the largest in point of size. The largest turbines so far are the 10,800 h.p. turbines of the Cedars Rapids (Canada) Mfg. and Power Co., which run at 56.6 r.p.m. under a head of 30 ft. The rated diameter is 143 in., but the maximum diameter (see Fig. 40) is 17 ft. 8 in. The runner weighs 160,000 lb., the revolving part of the generator and the shaft, 390,000 lb., while the suspension bearing weighs 300,000 lb. The total weight of the entire unit is 1,615,000 lb.

The largest runners in this country are those of the Mississippi River Power Co. at Keokuk, Ia. They develop 10,000 h.p. at 57.7 r.p.m. under a head of 32 ft. They are slightly smaller than those at Cedars Rapids. (See Figs. 42 and 47.) The I. P. Morris Co. built eight of these wheels and the Wellman-Seaver-Morgan Co. seven. These two concerns likewise built the Cedars Rapids turbines.

46. Efficiency.—The efficiency obtained from the average reaction turbine may be from 80 to 85 per cent. Under favorable conditions with large capacities higher efficiencies up to about 90 per cent. or more may be realized. For small powers or unfavorable conditions 75 per cent. is all that should be expected.

47. QUESTIONS

1. What was the origin of the Fourneyron turbine? What is the Jonval turbine? What was the origin of the Francis turbine?

2. What is the Swain turbine? What is the McCormick turbine? Why were they developed? What is the modern Francis turbine? Why is this name attached to all inward flow turbines at present?

3. Sketch the profiles of different types of modern turbine runners and explain why they are so built.

4. Why has the inward flow turbine superseded the outward flow turbine?

What classes of runners are there?

9. What are the different kinds of gates used for governing reaction turbines, and what are their relative merits?

10. What means are provided to save the penstock from water hammer, when the gates of a reaction turbine are quickly closed? How are the gates of a turbine operated?

11. What kinds of bearings are used for horizontal shaft turbines? For vertical shaft turbines? What means may be provided to take care of end thrust in either type?

12. What types of cases are used for turbines? What are the cheapest forms and what are the best? What are speed rings?

13. What is the purpose of a draft tube and how are they constructed?

14. What different factors may cause a turbine runner to wear out? Under what range of heads are reaction turbines now used?

15. What horsepower is developed by the most powerful turbine? What is the largest in point of size? Why is not the largest one also the most powerful? What efficiencies should be expected from reaction turbines?

48. General Principles.—All governors depend primarily upon the action of rotating weights. Thus the governor head in Fig. 66 is rotated by some form of drive so that its speed is directly proportional to that of the machine which it regulates. The higher the speed of rotation, the farther the balls stand from the axis, and the higher will the collar be raised on the vertical spindle. The collar in turn transmits motion to some element of the mechanism which effects the speed regulation.

Let W be the weight of each ball, $2KW$ that of the center weight, h the height of the "cone" in inches, x the ratio of the

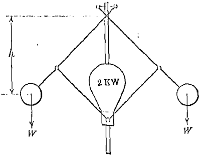


FIG. 66.—Governor head.

velocity of the collar to the vertical velocity of the balls, and N the revolutions per minute of the governor head. Also let the force which opposes the motion of the collar, due to the friction of the moving parts of the governor mechanism actuated by it, be denoted by $2fW$. Then the following equation may be found to hold:

$$N^2h = 35{,}200\,[1 + x(K \pm f)].$$

Considering the right hand member of the above as constant for the moment, it may be seen that for every value of h there must

ΔN and N be interpreted as the average speed, the coefficient of speed regulation is $\Delta N/N$. This coefficient may be reduced to a very small value by careful design of the governor. The essentials of a good governor are:

1. Close regulation or a small value of $\Delta N/N$.
2. Quickness of regulation.
3. Stability or lack of hunting.
4. Power to move parts or to resist disturbing forces.

To some extent certain of these requirements conflict with others

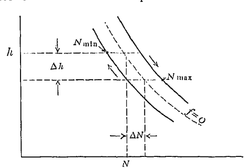


FIG. 67.

so that the final design is something of a compromise. Close regulation may be obtained by so proportioning the arms that x is a variable in such a way as to permit h to change but little for the different collar positions. Stability and power may be secured by making the center weight, $2KW$, sufficiently heavy. This weight is often replaced by a spring, which exerts an equivalent force. The importance of a large value of K is seen when we consider its relation to the friction. The latter changes sign according to the direction of motion, and may also change

[1] A constant speed or asynchronous governor could be constructed by so arranging it that h remained constant as the balls changed their position, but such a governor would lack stability as it might be in equilibrium with the collar in any position for a given speed. Then for a slight change in

The operation of the nozzles of impulse wheels or of the gates of reaction turbines requires a considerable force to be exerted. The governor head could not do this directly without being of prohibitive size and hence it does nothing more than set some relay device into action, the latter furnishing the power to operate the regulating mechanism.

49. Types of Governors.—There are two fundamental types of water wheel governors:

(*a*) Mechanical governors.

(*b*) Hydraulic governors.

With the first type the governor head causes some form of clutch to be engaged so that the gates are operated by the power of the turbine itself. This is the least expensive but has the disadvantage that the operation of the gates adds just that much more to a demanded load. The second type of governor costs more but is always used in the best plants. The governor head in this case merely operates a pilot valve which admits a liquid under pressure to one side or the other of a piston in a cylinder. This piston and cylinder is known as the servo-motor and operates the gates.

The liquid used to operate the servo-motor is stored under air pressure in a tank into which it is pumped. The power for operating the gates also comes from the turbine in this instance but it is spread over the entire period of operation instead of being concentrated just when the load is changing. Oil is commonly used as the working fluid and is very satisfactory except for its cost. Some effort, which is meeting with success, is being made to produce emulsions consisting principally of water but which will be similar to oil in its action. If water alone is used, it should be carefully filtered and circulated over and over again. Occasionally water has been used under penstock pressure and, of course direct from the penstock, but the grit and sediment in it is very bad for the operating parts of the governor.

50. The Compensated Governor.—It has already been pointed

ould continue to operate in the same direction under these
cumstances and would thus move far beyond the proper
int. Finally when the hydraulic conditions would readjust
emselves the governor would then be compelled to move back,
t would this time pass to the other side of the proper place.
us the governor would continually "hunt" and maintain a
nstant oscillation of flow in the pipe line.

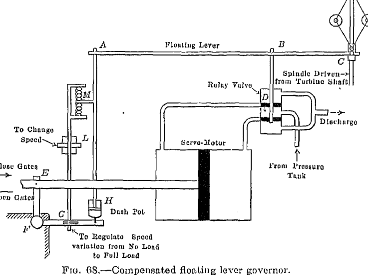
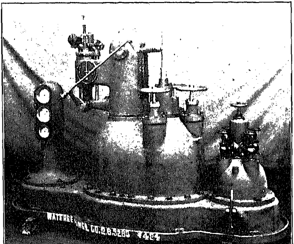


FIG. 68.—Compensated floating lever governor.

To prevent such action a waterwheel governor is usually
ompensated" so that it will slowly approach its proper place
d practically remain there. Such a governor is also said to be
ead beat." In Fig. 68 may be seen the essential features of
e floating-lever compensated governor. If, for example, the
eel speed increases, the balls of the governor raise the collar,
to which is attached the floating lever. The latter for the
oment pivots about A and through the link at B raises the relay
lve D. This action admits oil (or other fluid) to the left

rod at E, is rotated about F so the arm G is lowered. This pulls down the pivot A, which causes B to be lowered, thus closing the relay valve ports and stopping the motion. Thus the governor is prevented from over-travelling. Of course, if the gates have not been moved far enough, this action can be repeated.

The dash pot, H, will not cause the pivot A to be moved unless the governor acts quickly. If the governor changes slowly, there is little need for the compensating action and hence the dash pot does not then transmit the motion. But there is a



(*Courtesy of Allis-Chalmers Mfg. Co.*)
FIG. 69.—Hydraulic turbine governor.

second rod from G which is connected with the other vertical rod by springs at M. This will serve to stop the motion in such a case though it does not move A as much, since it has a shorter radius arm.

For a given speed of the governor head, and hence for a given position of the collar C, moving A will tend to shift the relay

can be altered by changing the radius of the connection at G. Considering B as fixed (as it must be if the relay valve is closed in both cases) it is evident that changing the amount of travel of A will change the amount of travel of the collar C. Remembering that different positions of collar C correspond to definite values of N, it is clear that changing the amount of travel of the collar C from no-load to full-load will vary the speed regulation.

Other adjustments that can be made to secure the proper degree of sensitiveness for the hydraulic conditions are to vary the springs at M and to change the speed of the dash pot.

One of the recent changes in governor construction for vertical type turbines is to mount the rotating weight on the turbine shaft itself. This eliminates any lost motion between the turbine and governor head.

51. QUESTIONS

1. With the usual type of governor, why must the speed vary to a slight extent from no-load to full-load? Which way does the speed change as the load increases? Why?

2. What qualification are essential in a good governor and how may they be obtained? What is the effect of friction on the operation of the governor?

3. Why is the speed range for a decreasing load different from that for an increasing load? What is the purpose of the center weight or the spring loading in governors?

4. What is a relay governor? Why is it necessary for water wheels? How is it operated?

5. What are the relative merits of different types of relay governors? What are the relative merits of the fluids used in hydraulic governors?

6. What is the compensated governor? Why is it necessary? Describe the action of one?

7. Describe the adjustments that can be made on a floating lever governor.

52. Equation of Continuity.—In a stream with steady flow (conditions at any point remaining constant with respect to time) the equation of continuity may be applied. This is that the rate of discharge is the same for all cross-sections so that $q = AV = av =$ constant, and in particular

$$q = A_1V_1 = a_1v_1 = a_2v_2 \tag{1}$$

53. Relation between Absolute and Relative Velocities.—The absolute velocity of a body is its velocity relative to the earth. The relative velocity of a body is its velocity relative to some other body which may itself be in motion relative to the earth. The absolute velocity of the first body is the vector sum of its velocity relative to the second body and the velocity of the second body. The relation between the three velocities u, v, V is shown

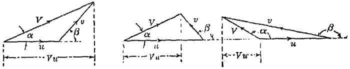


FIG. 70.—Relation between relative and absolute velocities.

by the vector triangles in Fig. 70. The tangential component of V is

$$V_u = V \cos \alpha = u + v \cos \beta \tag{2}$$

54. The General Equation of Energy.—Energy may be transmitted across a section of a flowing stream in any or all of the three forms known as potential energy, kinetic energy, or pressure energy.[1] Since head is the amount of energy per unit weight of water, the total head at any section

$$H = z + \frac{V^2}{2g} + \frac{p}{w} \tag{3}$$

[1] L. M. Hoskins, "Hydraulics," Chapter IV.

In flowing through the runner of a turbine the water gives up energy to the vanes in the form of mechanical work and a portion of the energy is lost in hydraulic friction and is dissipated in the form of heat. Thus the head lost by the water equals $h'' + h'$. And if suffixes (1) and (2) are restricted to the points of entrance to and discharge from the runner, equation (4) may be written

$$\left(z_1 + \frac{V_1^2}{2g} + \frac{p_1}{w}\right) - \left(z_2 + \frac{V_2^2}{2g} + \frac{p_2}{w}\right) = h'' + h' \qquad (5)$$

55. Effective Head on Wheel.—Obviously the turbine should not be charged up with head which is lost in the pipe line, so the

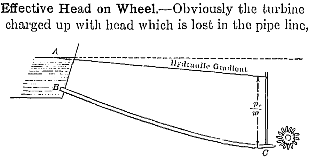


FIG. 71.—Effective head for tangential water wheel.

value of h should be the total fall available minus the penstock losses. Thus if Z is total fall available from head water to tail water, H' the head lost in the penstock or other places outside the water wheel, and h the net head actually supplied the turbine, we have

$$h = Z - H' \qquad (6)$$

The head supplied to the impulse wheel in Fig. 71 is the head measured at the base of the nozzle. Thus for the tangential water wheel

$$h = H_C = \frac{p_C}{w} + \frac{V_C^2}{2g} \qquad (7)$$

The reaction turbine, shown in Fig. 72, is able to use the total fall to the tail-water level by virtue of its employment of

h the total amount of the energy supplied to it. The kinetic ergy of the water at discharge from the mouth of the draft tube s a loss for which the runner and draft tube may be said to be ponsible in part, though some loss there is inevitable, but the uble is that the setting of the turbine, over which the turbine ilder has little control, limits the design of the draft tube and

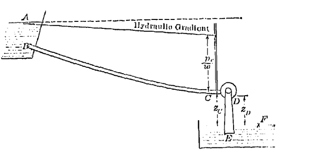


FIG. 72.—Effective head for reaction turbine.

nce the manufacturer may not be able to reduce this discharge s to a desired value. Two similar runners installed under ferent settings might yield different efficiencies because of this. onsequently turbine builders desire some method which will ake the measured efficiency of a runner independent of the nditions of the setting over which their designers have no ntrol. This second method is to charge up the turbine with all sses within the draft tube but to credit it with the velocity ad at the point of discharge. Thus

$$h = H_C - H_E = z_C + \frac{p_C}{w} + \frac{V_C^2}{2g} - \frac{V_E^2}{2g} \qquad (9)$$

It is believed that equation (8) is rational and scientifically rrect, but that equation (9) may be commercially more de- able.[1] In general the actual numerical difference between e values of h computed by these two methods will be small.

[1] For discussion on this point see "Investigation of the Performance of a

$$\text{Power} = wH = \text{pounds per second} \times \text{feet} \qquad (10)$$

In this expression H may be interpreted as in (3) or it may be replaced by h'' or any other head according to what is wanted.

But also power equals force applied times the velocity of the point of application. Thus

$$\text{Power} = Fu = \text{pounds} \times \text{feet per second} \qquad (11)$$

where F represents the total force applied.

Torque, T, equals $F \times r$ and angular velocity $\omega = \dfrac{u}{r}$

Since then $Fu = T\omega$ it is evident that

$$\text{Power} = T\omega = \text{foot pounds per second} \qquad (12)$$

Any of these three expressions for power may be used according to circumstances. While (11) is the most obvious to many, it will be found that in hydraulics (10) is usually more convenient.

(The following simplifications for horsepower of a turbine are convenient. Using the h of Art. 55,

$$\text{B.h.p.} = 62.5\ qhe/550 = qhe/8.8.$$

For estimations, the value of the efficiency may be assumed as 0.80 in which case our expression becomes h.p. $= qh/11$.

The word "efficiency" is always understood to mean total efficiency. It is the ratio of the developed or brake power to the power delivered in the water to the turbine based on the head h of Art. 55.

Reaction Turbine," by R. L. Daugherty, *Trans.* Am. Soc. of C. E., vol. 78, p. 1270 (1915).

It may be noted that it might be desirable under some circumstances to eliminate the draft tube losses altogether and compute the efficiency of the runner alone. This would necessitate the measurement of the head at D in order to take the difference between it and the head at C. The practical difficulty here is that, due to the turbulent and often rotary motion of the water at this point, it is impossible to measure the pressure with any degree of accuracy. Likewise the velocity head cannot be computed, since the actual velocity under the conditions of flow will not be equal to the rate of discharge

The difference between these two is due to hydraulic losses.

Volumetric efficiency is the ratio of the water actually passin through the runner to that supplied. The difference betwe these two quantities is the leakage through the clearance spac

The total efficiency is the product of these three. Thus

$$e = e_m \times e_h \times e_v.$$

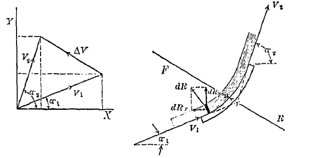


Fig. 73.

57. Force Exerted.—Whenever the velocity of a stream water is changed either in direction or in magnitude, a force required. By the law of action and reaction an equal ar opposite force is exerted by the water upon the body producir this change. This is called a dynamic force.

Let R be the resultant force exerted by any body upon t water and R_x and R_y be its components parallel to x and y axe Also let us here consider α as the angle made by V with the axis. The force exerted by the water upon the body will denoted by F. Its value may be found in either of the two wa shown below. The first depends upon the principle that t resultant of all the external forces acting on a body is equal the mass times the acceleration or $R = m\frac{dV}{dt}\cdot$ The second based upon the principle that the resultant of all the extern

the time rate of flow be dm/dt, where m denotes mass. Then in an interval of time dt there will flow past any section the mass $(dm/dt)\,dt$, which will be the amount considered. Thus

$$dR = dm\frac{dV}{dt} = \left(\frac{dm}{dt}.dt\right)\frac{dV}{dt} = \frac{dm}{dt}\left(\frac{dV}{dt}.dt\right)$$

Our discussion will be restricted to the case where the flow is steady in which case dm/dt is constant and equal to W/g. Therefore, since $(dV/dt)dt = dV$,

$$dR = \frac{W}{g}\,dV.$$

The summation of all such forces along the vane shown will give the total force. But, since integration is an algebraic and not a vector summation and in general these various elementary forces will not be parallel, it is necessary to take components along any axes. Thus

$$R_x = \frac{W}{g}\int_1^2 dV_x = \frac{W}{g}V_x\Big|_1^2$$

Now at point (1) the value of V_x is $V_1 \cos \alpha_1$ and at (2) it is $V_2 \cos \alpha_2$. Inserting these limits and noting from Fig. 72 that $V_2 \cos \alpha_2 - V_1 \cos \alpha_1 = \Delta V_x$, we have

$$R_x = \frac{W}{g}\,(V_2 \cos \alpha_2 - V_1 \cos \alpha_1) = \frac{W}{g}\,\Delta V_x$$

(b) *Force Equals Time Rate of Change of Momentum.*—Consider the filament of a stream in Fig. 74 which is between two cross-sections M and N at the beginning of a time interval dt, and between the cross-sections M' and N' at the end of the interval. Denote by ds_1 and ds_2 the distances moved by particles at M and N respectively. Let A_1 be the cross-section area at M, V_1 the velocity of the particles, and α_1 the angle between the direction of V_1 and any convenient x axis. Let the same letters with subscript (2) apply at N.

nce the change of momentum is the difference between the momentum of the part between N and N' and that of the part between M and M'. Noting that $wA_1ds_1 = wA_2ds_2$, since the flow is steady, the change in the x component of the momentum during dt is then

$$\frac{wA_1ds_1}{g}(V_2 \cos \alpha_2 - V_1 \cos \alpha_1).$$

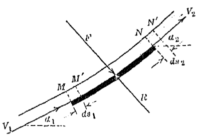


FIG. 74.

the rate of flow be denoted by W (lb. per sec.), then

$$wA_1ds_1 = Wdt$$

d the time rate of change of the x component of the momentum

$$\frac{W}{g}(V_2 \cos \alpha_2 - V_1 \cos \alpha_1).$$

us the x component of the resultant force is

$$R_x = \frac{W}{g}(V_2 \cos \alpha_2 - V_1 \cos \alpha_1) = \frac{W}{g}\Delta V_x.$$

This method has the advantage that it may be extended to e case where the flow is unsteady, if desired. In this event e two masses at the ends would be unequal and the momentum the portion from M' to N would be variable. In the case a series of vanes on a rotating wheel running at a uniform gular velocity the momentum of the water on any one vane

thod in (b) shows that the total force is independent of the h and depends solely upon the initial and terminal conditions. Since the force exerted by the water upon the object is equal l opposite to R, we have

$$F_x = \frac{W}{g}(V_1 \cos \alpha_1 - V_2 \cos \alpha_2) = -\frac{W}{g}\Delta V_x \qquad (13)$$

a similar manner the y component of F will be

$$F_y = \frac{W}{g}(V_1 \sin \alpha_1 - V_2 \sin \alpha_2) = -\frac{W}{g}\Delta V_y \qquad (14)$$

ce $F = \sqrt{F_x^2 + F_y^2}$ and $\Delta V = \sqrt{\Delta V_x^2 + \Delta V_y^2}$, value of the resultant force is

$$F = \frac{W}{g}\Delta V \qquad (15)$$

e direction of R will be the same as that of ΔV and the direction F will be opposite to it. It is because F and ΔV are in opposite ections that the minus sign appears the last members of equations (13) l (14). Note that ΔV is the *vector ference* between V_1 and V_2.

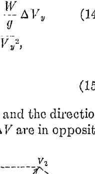


Fig. 75.

8. Force upon Moving Object.—e force exerted by a stream upon object may be computed by the ations of the preceding article, ether the object is stationary or mov-. The principal difference is that in latter case the determination of ΔV may be more difficult. us in Fig. 75 assume the initial velocity of the stream V_1, velocity of the object u_1, the angle between them α_1, and shape of the object to be given. The relative velocity v_1 be determined by the vector triangle. Its direction is also ermined by this triangle and is not necessarily the same as t of the vane or object struck by the water. But the direc- of the relative velocity of the water leaving is determined

v_1. Then knowing a_2, which is not necessarily equal to a_1 however, the magnitude and direction of V_2 can be computed from the vector triangle. The ΔV desired is the vector difference between V_1 and this V_2.

In case the stream is confined so that its cross-section is known, the magnitude of v_2 may be computed directly from the equation of continuity.

The remaining difficulty is the one of determining the amount of water acting upon the body per unit time. The rate of discharge in the stream flowing upon the object is A_1V_1 so that $W = wA_1V_1$. But this may not be the amount of water striking the object per second. For instance if the object is moving in

FIG. 76.—View showing action of jet on several buckets.

the same direction as the water and with the same velocity, it is clear that none of the water will strike it. The amount of water which will flow over any object is proportional to the velocity of the water relative to the object itself. If we denote by W' the pounds of water striking the moving object per second, and by a_1 the cross-section area normal to v_1, then $W' = wa_1v_1$.

If we consider a wheel with a number of vanes acted upon by the water, the above is true for one vane only. The reason that

nozzle to the wheel remains constant. If one vane s water than is discharged from the nozzle in any given erval and yet the wheel as a whole uses the entire amount , it means that the water must be acting upon more than ket at the same instant. This is shown in Fig. 76.

orque Exerted.—When a stream flows through a turbine n such a way that its distance from the axis of rotation unchanged, the dynamic force can be computed from the s of Art. 57. But when the radius to the stream varies, feasible to compute a single resultant force. And, if it would then be necessary to determine the location of its action in order to compute the torque exerted by it. ve find the total torque directly by other means.

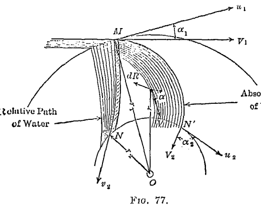


FIG. 77.

damental proposition of mechanics is that the time rate of of the angular momentum (moment of momentum) of tem of particles with respect to any axis is equal to the of the resultant external force on the system with respect ame axis.[1]

g. 77 let MN represent a vane of a wheel which may bout an axis O perpendicular to the plane of the paper. nters the wheel at M and, since the wheel is in motion

Only the tangential component of the velocity will appear in a moment equation, hence the angular momentum of this cylinder of water will be mass $\times$ radius $\times$ tangential velocity or (dm/dt) $dt \times r \times V \cos \alpha$, and the time rate of change of momentum, which is equal to torque, will be

$$dT' = \left(\frac{dm}{dt} \cdot dt\right) \frac{d(rV \cos \alpha)}{dt} = \left(\frac{dm}{dt}\right) \cdot d(rV \cos \alpha).$$

In the case of steady flow (dm/dt) is constant and equal to W/g and

$$T' = \frac{W}{g} \int_1^2 d(rV \cos \alpha).$$

Integrating between limits we have the value of the torque exerted by the wheel upon the water, or by changing signs, the value of the torque T exerted by the water upon the wheel. Thus

$$T = \frac{W}{g} (r_1 V_1 \cos \alpha_1 - r_2 V_2 \cos \alpha_2) \tag{16}$$

Representing the tangential component of the velocity of the water, often called the "velocity of whirl," by V_u, since it is in the direction of u, we have

$$T = \frac{W}{g} (r_1 V_{u1} - r_2 V_{u2}) \tag{17}$$

It is immaterial in the application of this formula whether the water flows radially inward, as in Fig. 76, radially outward, or remains at a constant distance from the axis. In any event r_1 is the radius at entrance and r_2 is that at exit.

A shorter method of proving the above is analogous to method (b) of Art. 57. During an interval of time dt the wheel has received angular momentum at M of $dmr_1V_1 \cos \alpha_1$ and given up angular momentum at N' of $dmr_2V_2 \cos \alpha_2$, assuming the flow to be steady. And, since $dm = (W/g)dt$ for steady flow, the time rate of change of angular momentum is $(W/g)(r_1V_1 \cos$

$r/g)V_2 \cos \alpha_2$ at radius r_2. Taking the moments of these two ngential forces, we get equation (16) at once.

60. Power and Head Delivered to Runner.—If the flow is eady and the speed of the wheel uniform, an expression for the ower developed by the water is readily obtained. From Art. 56

$$\text{Power} = Wh'' = T\omega.$$

sing the value of T given by (16) and noting that $r\omega = u$,

$$\text{Power} = Wh'' = \frac{W}{g}\left(u_1 V_1 \cos \alpha_1 - u_2 V_2 \cos \alpha_2\right) \qquad (18)$$

This is the power actually developed on the runner by the ater. It is analogous to the indicated power of a steam engine. he power output of the turbine is less than this by an amount qual to the friction of the bearings and other mechanical losses, ich as windage or the disk friction of a runner in water in the earance spaces.

Eliminating the W from the equation above we have the head tually utilized by the runner. Thus

$$h'' = e_h h = \frac{1}{g}\left(u_1 V_{u1} - u_2 V_{u2}\right) \qquad (19)$$

s just seen, the hydraulic efficiency is equal to h''/h. The net ead h supplied to the turbine is used up in the following ways:

$$h = h'' + k\frac{V_2^2}{2g} + m\frac{V_2^2}{2g} + k''\frac{V_1^2}{2g} \qquad (20)$$

f these items h'' is the head converted into mechanical work, the cond term represents the energy dissipated in the form of heat ne to internal friction and eddy losses within the runner, the ird term is the kinetic energy lost at discharge, and the fourth rm represents the loss in the nozzle of a Pelton wheel or the se and guides of a reaction turbine. The factor m in the bove may be unity in the case of an impulse wheel or a reaction rbine without a diverging or proper draft tube. For a reac- on turbine with an efficient draft tube it will be less than unity.

The substitution of these values gives us

$$\left(z_1 + \frac{v_1^2 - u_1^2}{2g} + \frac{p_1}{w}\right) - \left(z_2 + \frac{v_2^2 - u_2^2}{2g} + \frac{p_2}{w}\right) = h'$$

This equation serves to establish a relation between po (1) and (2). If the wheel is at rest u_1 and u_2 become zero, v_1 v_2 become absolute velocities and equation (21) becomes the eq tion of energy in its usual form as in (4).

62. Impulse Turbine.—The following numerical solutio given to illustrate the application of the foregoing princi This impulse turbine is of the outward flow type known as

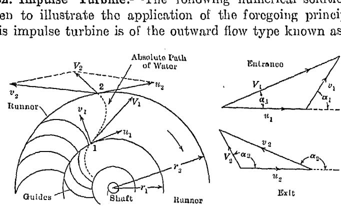


FIG. 78.—Outward flow turbine.

Girard turbine. Obviously the direction of flow makes no ference in the theory.

By construction, $\alpha_1 = 18°$, $\beta_2 = 165°$, $r_1 = 2.0$ ft., $r_2 = 2$ The hydraulic friction loss in flow through the runner wi taken as proportional to the square of the relative velocity so $h' = k\frac{v_2^2}{2g}$, where k is an empirical constant. Assume $k =$ Suppose $h = 350$ ft., $N = 260$ r.p.m., $q = 100$ cu. ft. per sec Find relative velocity at entrance to runner, relative vel and magnitude and direction of absolute velocity at exit runner, head utilized by wheel, hydraulic efficiency, losses, the horsepower. (See Fig. 77.)

it is an impulse turbine the pressure throughout the runner will be atmospheric. Thus $p_1 = p_2$.

Equation (21) then becomes

$$(1 + k)v_2^2 = v_1^2 + u_2^2 - u_1^2$$
$$1.4\ v_2^2 = 9910 + 4624 - 2960 = 11{,}574$$
$$v_2 = 90.9 \text{ ft. per second.}$$

By trigonometry $V_2 = 30.6$ ft. per second, $\alpha_2 = 130°$

$V_{u1} = V_1 \cos \alpha_1 = 150 \times 0.951 = 143$

$V_{u2} = u_2 + v_2 \cos \beta_2 - 68.0 = 90.9 \times 0.966 = -19.7$

(Also $V_{u2} = V_2 \cos \alpha_2 = 30.6 - \times (-0.639) = -19.7$)

$$h'' = \frac{1}{g}(u_1V_{u1} - u_2V_{u2})$$
$$= \frac{1}{32.2}(54.4 \times 143 + 68 \times 19.7) = 283 \text{ ft.}$$

Hydraulic efficiency $= h''/h = 283/350 = 0.81$.

Hydraulic friction loss $= k\dfrac{v_2^2}{2g} = 0.4\dfrac{8270}{64.4} = 51.3$ ft.

Discharge loss $= \dfrac{V_2^2}{2g} = \dfrac{940}{64.4} = 14.6$ ft.

Power $= \dfrac{Wh''}{550} = \dfrac{100 \times 62.5 \times 283}{550} = 3220$ h.p.

63. Reaction Turbine.[1]—Another numerical case will be given to illustrate the application of the foregoing principles to the reaction turbine. The turbine used here is the Fourneyron or outward flow type, though the theory applies to any type.

By construction, $\alpha_1 = 18°$, $\beta_2 = 165°$, $r_1 = 2.0$ ft., $r_2 = 2.5$ ft., $A_1 = 1.36$ sq. ft., $a_2 = 1.425$ sq. ft. Assume $k = 0.2\left(h' = k\dfrac{v_2^2}{2g}\right)$.

[1] See Art. 8. If the area a_2 is made small enough the wheel passages will be completely filled with water under pressure. We then have a reaction turbine. Note that

$V_1 = q/A_1 = 164.5/1.36 = 121$ ft. per second.
$v_2 = (A_1/a_2)\ V_1 = 115.5$ ft. per second.

For the above r.p.m. $u_1 = 110$ ft. per second, $u_2 = 137.5$ ft. per second.

$V_{u1} = V_1 \cos \alpha_1 = 115.$
$V_{u2} = u_2 + v_2 \cos \beta_2 = 137.5 - 115.5 \times 0.966 = 26.0.$
$h'' = \dfrac{1}{g}\,(u_1V_{u1} - u_2V_{u2}) = \dfrac{1}{32.2}\,(110 \times 115 - 137.5 \times 26) =$ 282 ft.
Hydraulic efficiency $= 282/350 = 0.805.$
Hydraulic friction loss $= k\dfrac{v_2^2}{2g} = 0.2\dfrac{13350}{64.4} = 41.5$ ft.
By trigonometry $V_2 = 41$ ft. per second.
Discharge loss $= \dfrac{V_2^2}{2g} = \dfrac{1680}{64.4} = 26$ ft.

Since v_2 is determined by the area a_2 we do not have the use for equation (21) that we did in the case of the impulse turbine. By it, however, we can compute the difference in pressure between entrance to and discharge from the runner. Thus from (21), taking $z_1 = z_2$,

$$\frac{p_1}{w} - \frac{p_2}{w} = \frac{(1 + k)v_2^2 - u_2^2 - v_1^2 + u_1^2}{2g} = 122 \text{ ft.}$$

(If the turbine discharges into the air then $\dfrac{p_2}{w} = O$ and $\dfrac{p_1}{w} = 122$ ft.) This pressure difference may also be computed from equation (5).

$$\text{Power} = \frac{Wh''}{550} = \frac{62.5 \times 164.5 \times 282}{550} = 5270 \text{ h.p.}$$

64. Effect of Different Speeds.—If a wheel is run at different speeds under the same head, the quantities v_1, v_2, V_2, α_2, h'', efficiency, and power all vary. In Fig. 79 may be seen the

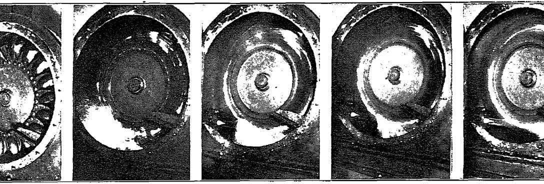
0°. The value of V_2 decreases to a minimum and then increases again.

From equation (20) it may be seen that, other losses being equal, the maximum efficiency would be obtained when the discharge loss is a minimum. It can be seen that V_2 is very small when either $v_2 = u_2$ or $\alpha_2 = 90°$. A means of computing the speed necessary for this will be given later. Neither of these gives the actual mathematical minimum but they are very close to it.

The torque exerted on the wheel by the water may be seen to decrease as the wheel speed increases. In equation (17) W and V_{u1} are practically constant, though they vary slightly in the case of the reaction turbine. But V_{u2} continuously increases algebraically. It has its maximum negative value when the wheel is at standstill, it is zero when the speed is such that $\alpha_2 = 90°$, and it attains its maximum positive value when the turbine is at run-away speed. This is the maximum speed which the wheel can reach under a given head and is attained when all external load is removed. Under these circumstances the torque exerted by the water is just sufficient to overcome the mechanical losses of the turbine. The run-away speed of the wheel is thus strictly limited by hydraulic conditions.

In the ideal case the maximum possible speed of the Pelton wheel would be when the velocity of the buckets equalled the velocity of the jet. But under these conditions ΔV of equation (15) would equal zero. Consequently the wheel must run at a speed somewhat less than this as some power is required to overcome the mechanical losses at this speed. For the impulse wheel the maximum value of ϕ usually attained is about 0.80 at run-

[1] These photos also show the needle in the center of the jet. The piece at the side of the buckets toward the lower right hand side of the case is the "stripper," its function being to deflect water that might otherwise be carried around with the wheel up into the upper part of the case. The buckets pass through this with a relatively small clearance. A close inspection of the views of the wheels and the water leaving it will give one a fair



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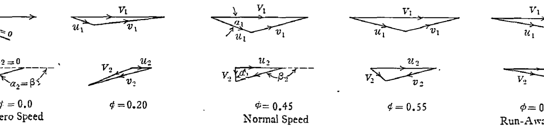


FIG. 79.—42″ Impulse wheel at different speeds under same head.

0–0, the water filling the vessel will tend to rotate at the same speed with it and we will have a forced vortex.

The pressure within this body of water will then vary as shown by the curve CD. The law of variation may be found as follows

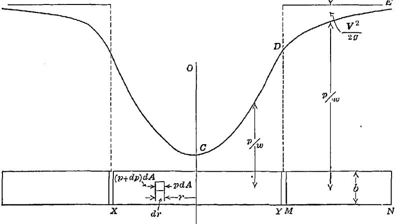


Fig. 80.—Forced and free vortices.

Consider an elementary volume, whose length along the radius is dr and whose area normal to this is dA, and which rotates at an angular velocity ω at radius r. The difference in the pressures on the two faces, which is the resultant force acting, is equal to $dp \times dA$, and the acceleration of the mass is $\omega^2 r$, directed toward the axis of rotation. Thus

$$dpdA = (wdA\ dr/g)\ \omega^2 r$$
$$dp = (w\omega^2/g)\ rdr$$
$$p = (w\ \omega^2/g)\ r^2/2 + \text{constant.}$$

To find the value of the constant of integration, let p_o be the pressure when r equals zero. Thus the constant is equal to p_o, and

vessel is open, but with sides high enough so that the water cannot overflow, the surface of the water will become a paraboloid, since the pressure variation along the radius is the same whether the water be confined or not.

This equation is really a special case of equation (21) with v_1 and v_2 equal to zero, since there is no flow of water. If water flows then equation (21) must be employed. Flow may occur in either direction. It may be noted that the energy of the water is not constant along the radius, as both the pressure and the velocity of the water increase. This is possible because, due to the action of external forces, energy is being delivered to (or taken from) the water.

An important application of equations (21) and (22) is in connection with the centrifugal pump. The vessel XY of Fig. 78 may be said to be a crude illustration of the impeller of such a pump. But the equations are also of value in determining the conditions of flow through turbine runners, of either the impulse or reaction type.

66. Free Vortex.—A *free* vortex is produced when a liquid rotates by virtue of its own angular momentum, previously derived from some source, and is free from the action of external forces. Thus in Fig. 80, if the rotating vessel XY is surrounded by a stationary vessel MN into which the water can pass from XY, the water will still tend to rotate and, neglecting friction, we will have a free vortex.

The pressure within the free vortex will vary as shown by the curve DE. The law of variation may be found as follows: Since no external forces are applied, the resultant torque exerted is zero, and hence the angular momentum is constant (Art. 59). Since angular momentum is proportional to $rV \cos \alpha$ or rV_u, it follows that

$$rV \cos \alpha = rV_u = \text{constant} \tag{23}$$

the value of the constant being the value obtained from the numerical value of these factors at the point of entrance.

that

$$V^2 = V_u^2 + V_r^2 \tag{25}$$

Since no energy is delivered to (or taken from) the water, the lue of the effective head H must remain constant. Thus

$$H = z + \frac{V^2}{2g} + \frac{p}{w} = \text{constant},$$

e value of this constant being determined by the total head of e water initially. Taking the datum plane such that $z = 0$, e have

$$\frac{p}{w} = H - \frac{V^2}{2g} = H - \frac{V_u^2}{2g} - \frac{V_r^2}{2g} \tag{26}$$

The flow of the water may be in either direction. Actual fric- on losses will modify the resulting values of the pressure and so of V_u, but cannot alter V_r. If b is constant, V_u and V_r vary the same proportion, neglecting friction, so that α is constant d the path of the water is the equi-angular or logarithmic iral.

The free vortex is found in the casing surrounding the im- ller of some types of centrifugal pumps. It is also found in e water in a spiral case approaching a turbine runner, and e above equations have many applications.

For example equations (23), (24), and (25) show that the locity of the water varies inversely as the radius of curvature its path. Hence if the vanes of turbine runners are so aped that the stream lines have sharp curvatures, the velocity the water will be excessive and, from equation (26), it may seen that the pressure will be reduced. This may result the pressure becoming so low that erosion will result, mentioned in Art. 45. For the same reason it is undesirable let the water discharged from a turbine runner flow direct ward the axis, as in the pure radial inward flow turbine. r if the water leaving the runner had any "whirl" this would crease as the axis was approached and, according to the

67. Theory of the Draft Tube.—The flow of water through a draft tube is no different in principle from the flow through any

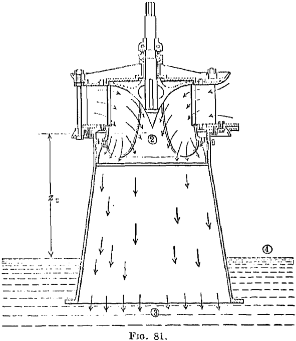


FIG. 81.

other conduit and hence Bernoulli's theorem, otherwise known as the general equation of energy of Art. 54, may be applied to it. Equation (4) however has been stated only for the case of steady flow and for the present purpose we are concerned with any condition of flow that may exist. Hence we shall add another term called the acceleration head, which is the head necessary to accelerate the velocity of the water when the rate of discharge is changed by the action of the governor. Let this head be denoted by h_{acc}, while the loss of head in friction, H', is divided

$$H_4 = 0 + 0 + \frac{p_a}{w}$$

where p_2 denotes absolute pressure and p_a denotes atmospheric pressure. Then

$$H_2 - H_4 = H' + h_{acc}$$

Or

$$z_2 = \frac{p_a}{w} - \frac{p_2}{w} - \frac{V_2^2}{2g} + h_f + \frac{V_3^2}{2g} + h_{acc} \tag{27}$$

The solution of this equation will give the allowable height of the turbine runner above the tail-water level. Or the equation can equally well be used to determine the pressure for any given elevation. In the above,

$\frac{p_a}{w}$ is governed by the altitude and local variations but is approximately equal to 34 ft. of water.

$\frac{p_2}{w}$ cannot be less than the vapor pressure of the water at that temperature as determined from the steam tables and should be from at least 2 to 4 ft. of water more.

V_2 is a function of the design and type of the runner. The higher the capacity and speed of the type the higher will be the value of V_2. It is also a function of the head under which the turbine runs, because all velocities vary as the square root of the head. Also if α_2 is not 90°, the value of V_2 will be greater than q divided by draft tube area.

h_f depends upon the construction of the draft tube. Ordinarily it may be assumed as about 15 to 25 per cent. of $V_2^2/2g$.

V_3 is controlled by the setting of the plant for that fixes the allowable length of the draft tube. It is also a function of the construction of the draft tube and the head under which the turbine operates.

h_{acc} is determined by the action of the governor and it may be either plus or minus in value. The negative value is the one to use in the above equation.

also require a lower setting under a higher head because of change in this same item.

If the turbine is set higher than the limiting value, as d mined by this equation, the efficiency of conversion in draft tube will be lost due to the vaporization and subseq recondensation of the water. Also corrosion of the runner take place because of the liberation of oxygen. Again if height is very close to the allowable limit for steady flow sudden closure of the turbine gates by the governor may caus pressure at discharge from the runner to drop to such a low v that the water vaporizes. But an instant later the water surge back up the draft tube, striking the runner a decided l

68. QUESTIONS AND PROBLEMS

1. How is the effective head to be measured on the Pelton wheel a the reaction turbine? Why are two values possible in the latter What are the definitions of the various efficiencies that may be dealt

2. What is the procedure for computing the force exerted by a s upon a moving object? What are the reasons for the difference be W and W'?

3. What becomes of the total energy supplied in the water to the w As the speed of a wheel varies, under a constant head, the torque ex on it, and consequently its power, varies. Since the power supplied water is constant, what becomes of the difference between the two?

4. What are the fundamental differences between the solution of the lem of the impulse wheel and of the reaction turbine?

5. As the speed of a wheel changes how do V_2 and α_2 vary? Of significance is this? What limits the maximum speed of a Pelton under a given head?

6. What is a forced vortex? How does the pressure vary in it? examples of it are found?

7. What is a free vortex? How does the velocity vary in it? Hov the pressure vary? What common examples of this are found?

8. What conclusions can one draw from the equations for the free that have an important practical application?

9. Derive the equation for the maximum allowable height of a tu runner and discuss the items that affect this value?

10. What is the effect of the action of the governor upon the

normal to jet, (c) magnitude and direction of total force.
ming the terminal velocity to be reduced to 80 ft. per second, rs remaining the same.

Ans. (a) 211 lb., (b) 365 lb., (c) 422 lb. at 60° with V_1.
(a) 254 lb., (b) 293 lb., (c) 388 lb. at 49° 08′.

roblem (13) assuming the angle of deflection to be 180°. o does the angle make in the magnitude of each force? What ere in the effect of friction in each instance?

Ans. (a) 844 lb., (b) 0, (c) 844 lb. at 0° with V_1.
(b) 760 lb.

the vane in problem (14) moves in the same direction as the city u, and that friction loss is such that $v_2 = 0.8v_1$. When O, 30, 44.4, 70 and 100 ft. per second, find: (a) Values of the r second striking the vane, (b) values of absolute velocity at alues of the force exerted.

Ans. (a) 136.3, 95.3, 75.7, 40.8, and 0 lb. per second.
(b) $-80, -26, 0, +46$, and $+100$ ft. per second.
(c) 760, 372, 234, 68.5, and 0 lb.

oblem (15) if the jet is upon a wheel equipped with similar the power delivered to the shaft at each speed. What be- ifference between this and the power of the jet?

Ans. 760, 532, 422, 228, and 0 lb.
0, 29.0, 34.0, 29.0, and 0 h.p.

rbine runner, $V_1 = 70$ ft. per second, $V_2 = 20$ ft. per second, = 3 ft., $\alpha_1 = 20°$, $\alpha_2 = 80°$, and $W = 300$ lb. per second. (a) xerted upon the wheel. (b) If $u_1 = 50$ ft. per second, find

Ans. (a) 1128 ft. lb., (b) 51.3 h.p.

rbine runner, $V_1 = 70$ ft. per second, $V_2 = 20$ ft. per second, , $p_2/w = -25$ ft. Assume friction loss $(kv_2{}^2/2g)$ in flow r as 5.78 ft. and that there is no difference in elevation. (a) ized by runner, (b) If $W = 300$ lb. per second, find the power.

Ans. (a) 94.2 ft., (b) 51.3 h.p.

impulse turbine in Art. 62 it will be found that $v_2 = u_2$ when per second. Find the r.p.m., efficiency, losses, and horse- are with values given in Art. 62.

Ans. 326 r.p.m., $e = 0.845$, 3365 h.p.

reaction turbine in Art. 63 it will be found that $\alpha_2 = 90°$ if or second. At that speed the rate of discharge will be found iven later) to be 159 cu. ft. per second. Find the r.p.m., es, and horsepower. Compare with values given in Art. 63.

Ans. 412 r.p.m., $e = 0.852$, 5380 h.p.

the best r.p.m. of the impulse turbine with the best r.p.m.

to the speed ring vanes, the outer radius of which is 7 ft.? (b) If the height of the vanes at this point is 5 ft., find the radial component of the velocity if the turbine discharges 900 cu. ft. per second. (c) What should be the angle of the speed ring vanes at this point? (d) What should be the angle of entrance to the turbine guide vanes, if the radius is 6 ft., and the height is 3 ft.? *Ans.* (a) 12.85 ft. per second, (b) 4.09 ft. per second.

23. A turbine running under a head of 200 ft. is of such a design that $V_2^2/2g = 7$ per cent. of h and $V_3^2/2g = 1$ per cent. of h. If the minimum pressure allowable is 3 lb. per sq. in., what is the maximum height the runner may be set above the tail-water level, assuming the draft tube loss to be 25 per cent. and the maximum negative acceleration head to be 50 per cent. of $V_2^2/2g$? What will be the result if this same turbine is used under a head of 50 ft.? *Ans.* 11.6 ft., 23.2 ft.

24. A turbine running under a head of 50 ft. is of such a design that $V_2^2/2g = 20$ per cent. and $V_3^2/2g = 2$ per cent. of h. If the minimum pressure allowable is 3 lb. per sq. in., what is the allowable height the runner may be set above tail-water level, assuming the draft tube loss to be 25 per cent. of $V_2^2/2g$ and the maximum negative acceleration head to be 50 per cent. of $V_2^2/2g$? Compare with second part of preceding problem.

Ans. 15.6 ft.

THEORY OF THE TANGENTIAL WATER WHEEL

69. **Introductory.**—The tangential water wheel has been assed as an impulse turbine with approximately axial flow. e term tangential is applied because the center line of the jet tangent to the path of the centers of the buckets. In this ticle the assumption will therefore be made that $\alpha_1 = 0°$ and at $r_1 = r_2$. It will be shown later that these assumptions are t entirely correct.

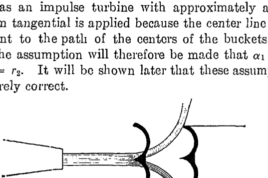


FIG. 82.

If the angle α_1 be assumed equal to zero then u_1 and V_1 are the same straight line and $v_1 = V_1 - u_1$. The conditions at it from the buckets are shown in Fig. 82. In applying equation we desire to find only the component of the force tangential to e wheel since that is all that is effective in producing rotation. erefore we shall find only the component of ΔV along the direcn of u_1. Thus, if F here denotes tangential force,

$$F = \frac{W}{g}(V_1 - V_2 \cos \alpha_2)$$

$$= \frac{W}{g}(V_1 - u_1 - v_2 \cos \beta_2)$$

$$F = \frac{W}{g}\left(1 - \frac{\cos \beta_2}{\sqrt{1 + k}}\right)(V_1 - u_1) \qquad (28)$$

nore exact value for the force exerted may be found in Art. 72. e above is only an approximation.

Multiplying the force given above by the velocity of its point ıpplication, we have the power developed. Thus

$$P = Fu_1 = \frac{W}{g}\left(1 - \frac{\cos \beta_2}{\sqrt{1 + k}}\right)(V_1 - u_1)u_1 \qquad (29)$$

e power input to the wheel, including the nozzle, is Wh, where s determined as in Art. 55. The power in the jet is $WV_1^2/2g$ d is less than the former by the amount lost in friction in the zle.

Equation (28) is the equation of a straight line. It shows that s a maximum when u_1 is zero and that it decreases with the ed until it becomes zero when $u_1 = V_1$. Equation (29) is the ıation of a parabola. It shows that the power is zero when $= 0$ and again when $u_1 = V_1$. The vertex of the curve, which es the maximum power and hence the maximum efficiency, is nd when $u_1 = 0.5V_1$. Since in reality both of these curves altered somewhat, when all the factors are considered, some hese statements require modification.

The actual speed for the highest efficiency has been found by t to be such that $\phi_e = 0.45$ approximately, while the value of efficiency is about 80 per cent. Both of these values vary newhat with the design of the wheel and the conditions of use. t one can approximately compute the bucket speed and the ver of any impulse wheel, provided the head and size of are known. The bucket speed $u_1 = \phi\sqrt{2gh}$, while the ocity of the jet $V_1 = c_v\sqrt{2gh}$. For a good nozzle with full ening, if equipped with a needle, the coefficient of velocity uld be about 0.98. Thus the rate of discharge is determined. he diameter of the wheel is known, or assumed as a function he size of the jet, the rotative speed can be computed.

at B_1 and begins to cut off the water from the preceding bucket C_1. When the bucket reaches the position B_2 the last drop of water will have been cut off from C_2, but there will be left a portion of the jet, $MPXY$, still acting upon it. The last drop of water X will have caught up with this bucket when it reaches position C_3. Thus while the jet has been striking it the bucket has turned

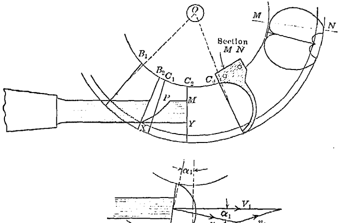


Fig. 83.

through the angle B_1OC_3. The average value of α_1 will be taken as the angle obtained when the bucket occupies the mean between these two extreme positions. It is evident that position C_3 will depend upon the speed of the wheel, and that the faster the wheel goes the farther over will C_3 be. Thus the angle α_1 decreases as the speed of the wheel increases. The variation in the value of α_1 as worked out for one particular case is shown in Fig. 85.

71. The Ratio of the Radii.—It is usually assumed that $r_1 = r_2$. However inspection of the path of the water in Fig. 84 (*a*) will show that when the bucket first enters the jet r_2 may be less than

vly compared with the jet velocity the value of x will be less n when the wheel is running at a higher speed. This may be ified by actual observation. When the wheel is running at its per speed it is probably true that x is very nearly equal to ty. In any case the variation of the value of x from unity not be very great.

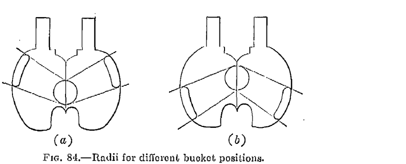


FIG. 84.—Radii for different bucket positions.

72. Force Exerted.—The force acting on the wheel may be ermined by the principles of Art. 58, but, if the radii are not ual it will not be convenient to use equation (15) on account of difficulty of locating the line of action of the force. However we can use equation (17) and by it determine a force at radius r_1 which shall be the equivalent of the real force. viding (17) by r_1 and letting F denote tangential force we ain

$$F = \frac{W}{g}(V_{u1} - xV_{u2})$$

$$V_{u1} = V_1 \cos \alpha_1$$

$$V_{u2} = u_2 + v_2 \cos \beta_2.$$

equation (21)

$$(1 + k)v_2^2 = v_1^2 + u_2^2 - u_1^2.$$

trigonometry

$$v_1^2 = V_1^2 - 2V_1u_1 \cos \alpha_1 + u_1^2$$

$$g \Big[\qquad \sqrt{1+k} \qquad \Big] \tag{30}$$

quation (30) is a true expression for the force exerted. No reat error is involved, however, by taking $x = 1.0$. If that is one the expression under the radical becomes the value of v_1 nd may be found graphically. For the sake of simplicity and ase in computation α_1 may be taken equal to zero and the equa- on then reduces to (28), but an exact value of F will not be ob- ained. There is little excuse for taking $k = 0$, as most writers o, for equation (28) is not simplified to any extent and the re- ılts are entirely incorrect.

73. Power.—With F as obtained from (30) the power is given y Fu_1. We may also compute h'' and obtain the power by ultiplying by W.

Since $h'' = \frac{1}{g} u_1 (V_{u1} - xV_{u2})$ it is evident that the expression for $''$ is the same as (30) if u_1 be substituted for W. Thus the expres- on for power has the same value no matter from which basis it is erived.

74. The Value of W.—W is the total weight of water striking ie wheel per second. It is obvious that the weight of water ischarged from the nozzle is

$$W = w\,A_1V_1.$$

nder normal circumstances all of this water acts upon the wheel. lowever for high values of the ratio u_1/V_1 a certain portion of ie water may go clear through without having had time to atch up with the bucket before the latter leaves the field of action. is apparent, for instance, that if the buckets move as fast as the t none of the water will strike them at all. For all speeds less ian that extreme case a portion of the water only may fail to act. 'hus referring to Fig. 83, it can be seen that if the wheel speed is igh enough compared to the jet velocity the water at X may not ave time to catch up with bucket C. The variation of W with

to the size of jet that may be used for a given wheel, as stated in Art. 30. For a given diameter of wheel, as the size of the nozzle is increased, larger buckets must be used and they must also be spaced closer together.

75. The Value of k.—The value of k is purely empirical and must be determined by experiment. If the dimensions of the wheel are known and the mechanical friction and windage losses are determined or estimated, then from the test of the wheel the horse-power developed by the water may be obtained. The value of k is then the only unknown quantity and may be solved

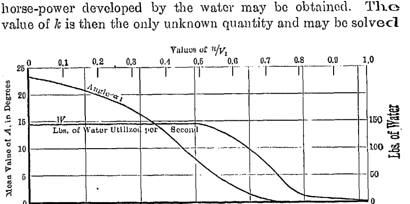


FIG. 85.—Values of α_1 and W for a certain wheel.

for. The value of k is probably not constant for all values of u_1/V_1. Some theoretical considerations, which need not be given here, have indicated that it could scarcely be constant and an experimental investigation has shown the author that k decreased as u_1/V_1 increased. For a given wheel speed however it is nearly constant for various needle settings unless the jet diameter exceeds the limit set in Art. 30. The crowding of the bucket then increases the eddy losses and would require a higher value of k.

The value of k may be as high as 2.0 but the usual range of values is from 0.5 to 1.5.

76. Constant Input—Variable Speed.—The variation of torque

given nozzle opening the horsepower output is fixed and constant. The horsepower output varies with the speed. It will be noticed that the maximum efficiency is attained at slightly higher speeds

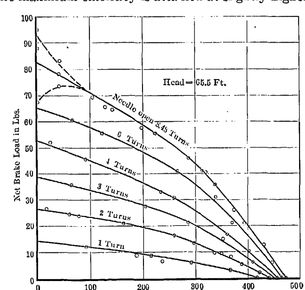


FIG. 86.—Relation between torque and speed.

for the larger nozzle openings than for the smaller. This is due, in part, to the fact that the mechanical losses, which are practically constant at any given speed, become of less relative importance as the power output increases.

Fig. 88 shows the variation of the different losses for a constant power input but a variable speed.[1]

77. Best Speed.—It is usually assumed that the best speed is the one for which the discharge loss is the least. As shown in Art. 64, the latter will be approximately attained either when $u_2 = v_2$ or when $\alpha_2 = 90°$. In the case of the impulse turbine the former

[1] The curves shown in this chapter are from the test of a 24-in. tangential

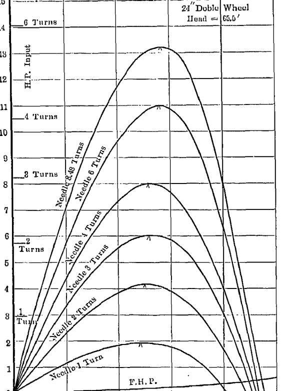


FIG. 87.—Relation between power and speed for different needle settings.

that, although the difference is not great, the above equation does not give the best speed. The hydraulic friction losses and

* L. M. Hoskins, "Hydraulics," Art. 198, Art. 208.

The speed of any turbine is generally expressed as $u_1 = \phi\sqrt{2gh}$. The coefficient of velocity of the nozzle will reduce the above

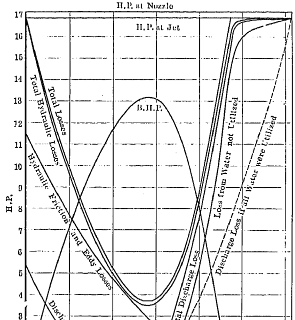


FIG. 88.—Segregation of losses for constant input and variable speed.

values slightly, so that the best speed is usually such that

$$\phi_e = 0.43 \text{ to } 0.47$$

78. Constant Speed—Variable Input.—The case considered in

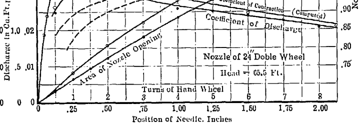


FIG. 89.—Nozzle coefficients and other data.

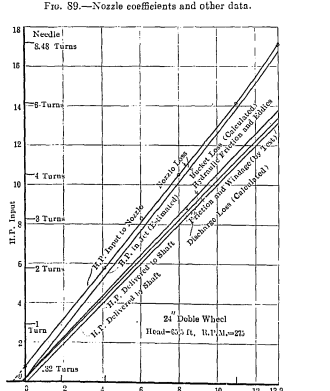


FIG. 90—Relation of input to output and segregation of losses for variable input and ...

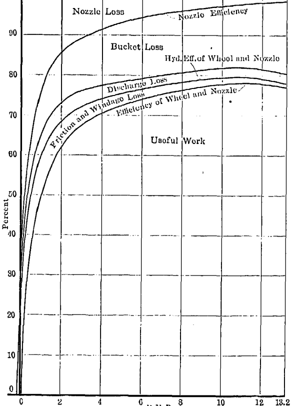


FIG. 91.—Efficiencies and per cent. losses at constant speed.

very nearly a straight line. Above six turns it bends up slightly because the wheel is then slightly overloaded.

The friction and windage was determined by a retardation run[1] and was assumed to be constant at all loads. The hydraulic losses were segregated by the theory already given (Art. 62). These results plotted in per cent. are shown in Fig. 91 and Fig. 92.

is really the proper one to use. But the important fact is that the angle is not zero and that it does vary. In similar manner the determination of the amount of water acting upon the wheel at speeds above normal, and the determination of the speed at which this waste of water begins, is difficult. But the consideration of the problem makes it clear why the curves for the force exerted are not straight lines, as may be seen in Fig. 86, and why the right-hand portion is steeper. In turn this explains why the actual power curves of Fig. 87 are distorted parabolas with the right-hand side much steeper than the left-hand side. Ideally the maximum speed of the wheel should be such that $u_1 = V_1$, but actually the run-away speed is such that $\phi = 0.80$ approximately. This is due to the fact that the proportion of the water acting on the wheel at higher speeds would become so small that the force exerted would be less than that required to overcome the bearing friction and windage loss.

The losses computed by theory and in part determined by experiment are shown in Fig. 88. If it were not for the waste of water mentioned above, the discharge loss from the buckets would be as shown by the dotted line. Actually the loss of energy in this water is shown by the solid curve to the left of this, while the discharge loss from the buckets is only the intercept between the latter curve and the one to its left.

The theory, as illustrated in Fig. 90, shows that for a wheel at the proper speed the principle loss of energy is in the buckets. This emphasizes the importance of close attention to the proper design of the latter. The theory also shows that the individual losses tend to follow straight line laws. This means that the relation between input and output is also a straight line. When the size of the jet becomes too large for the particular wheel, the bucket losses increase more rapidly and hence the curve bends upward at this point, as shown. The relation between input and output is not exactly a straight line for loads less than that for maximum efficiency, but it is nearly so. This is of interest be-

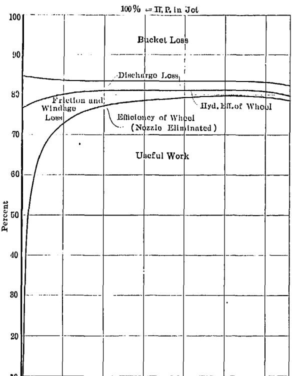


FIG. 92.—Efficiencies and per cent. losses at constant speed based upon power in jet.

together the hydraulic efficiency does not begin to drop off rapidly until very small nozzle openings are reached. The reason for this is that the vector velocity diagrams upon which the theory is based are independent of the size of the jet. The

whose coefficient of velocity = 0.97. Suppose the diameter of the jet from the nozzle = 6 in. Let this jet act upon a tangential water wheel of the following dimensions: Diameter = 6 ft., $\alpha_1 = 12°$, $\beta_2 = 170°$. Assume $k = 0.6$, $\phi = 0.465$, and assume bearing friction and windage = 3 per cent. of power input to shaft.

The problem of the pipe line is a matter of elementary hydraulics and a detailed explanation will not be given of the steps here employed. The coefficient of loss at B will be taken as

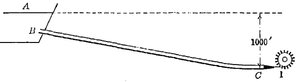


FIG. 93.

1.0, the coefficient of loss in the pipe will be assumed 0.03. The loss in the nozzle will be given by $\left(\frac{1}{c_v^2} - 1\right)\frac{V_1^2}{2g}$, where c_v = the coefficient of velocity and V_1 the velocity of the jet. If V_c = the velocity in the pipe then the losses will be

$$\left(1 + 0.03\frac{5000}{2.5}\right)\frac{V_c^2}{2g} + 0.063\,\frac{V_1^2}{2g}$$

Taking $H_A = 1000$ ft. and $H_1 = \frac{V_1^2}{2g}$ then by equation (4) we may solve for $\frac{V_c^2}{2g} = 1.38$ ft. or $\frac{V_1^2}{2g} = 861$ ft.

Thus $V_c = 9.42$ ft. per second and $V_1 = 235.5$ ft. per second.
Rate of discharge, $q = 4.62$ cu. ft. per second.

The pressure head at nozzle, $\frac{p_c}{w} = 914.5$ ft.

The wheel speed $u_1 = 0.465 \times 8.025\sqrt{915.88} = 113$ ft. per second.

be obvious.

Total head available,	$H_A = 1000$ ft.
Head at nozzle,	$H_c = 915.88$ ft.
Head in jet,	$H_1 = 861$ ft.
Head utilized by wheel,	$h'' = 757$ ft.

Total power available at A	$= 5250$ h.p.
Power at nozzle (C)	$= 4800$ h.p.
Power in jet	$= 4520$ h.p.
Power input to shaft	$= 3970$ h.p.
Power output of wheel	$= 3851$ h.p

Hydraulic efficiency of wheel	$= 0.878$
Mechanical efficiency of wheel	$= 0.970$
Gross efficiency of wheel	$= 0.852$
Efficiency of nozzle	$= 0.941$
Gross efficiency of wheel and nozzle	$= 0.801$
Efficiency of pipe line BC	$= 0.915$
Overall efficiency of plant	$= 0.733$

81. QUESTIONS AND PROBLEMS

1. With the simple theory of the tangential wheel what are the relations for torque and power as functions of speed? How may the speed and power of an impulse wheel be computed in practice?

2. What are the true conditions of flow in the Pelton water wheel and what assumptions are often made in order to simplify the theory?

3. When may a portion of the water discharged from a nozzle fail to act upon the wheel? Why? What changes in design will improve this condition?

4. Why is the relation between input and output at a constant speed and head not a straight line throughout its range? How does the hydraulic efficiency vary from no-load to full-load at constant speed? Why?

5. Suppose the dimensions of a tangential water wheel are: $\beta_2 = 165°$, $\phi = 0.45$, $k = 0.5$, and the velocity coefficient of the nozzle $= 0.98$. If the diameter of the jet $= 8$ in. and the head on the nozzle 900 ft., compute

ing an efficiency of 80 %, what will be the size of jet required, and what will be the approximate diameter and r.p.m. of the wheel?

82. Introductory.—The main purpose of this chapter is to explain the characteristics of reaction turbines. In turbine theory there are many variables and one must assume some of these and compute the rest, and, according to what is assumed as known, the theory presented by various indididuals will differ. Also there are matters of difference of detail. For instance one may assume the hydraulic friction losses through the entire turbine, including guides and runner, to be some function of the rate of discharge, while another will attempt to analyze these losses and compute them individually.

The turbine designer, desiring to obtain some definite performance, naturally assumes certain results and computes the dimensions necessary. For our present purpose, we shall do exactly the opposite of this and assume all the dimensions as known and endeavor to determine the characteristics of the given turbine.

83. Simple Theory.—A very simple theory is possible by assuming certain factors to be known as the result of experience. Thus, as in the case of the impulse wheel, the peripheral velocity of the runner may be represented as $u_1 = \phi\sqrt{2gh}$. And the speed at which the efficiency is a maximum is given by values of ϕ_e ranging from 0.55 to 0.90 according to the type of the runner as in Fig. 34, page 43. This differs from the Pelton wheel not only in the numerical values of ϕ_e but also in the much greater range that is possible.

The efficiency of the turbine may be assumed as from 80 to 90 per cent. according to the size and type of the runner, and hence the power may be computed if the rate of discharge is known. We here introduce another factor c such that $V_1 = c\sqrt{2gh}$. (This is really a velocity coefficient but there is no need to draw any distinction between it and the coefficient of discharge, since here the coefficient of contraction is unity.) It may be noted that in the reaction turbine the water is under pressure throughout its flow and hence the total energy of the water entering the

action of the wheel has no effect upon the velocity of the water from the nozzle. Thus the factor c is not only less than unity but it depends upon the design and type of the runner, and furthermore it varies with the speed of the latter. Because of this variation with the speed, we shall here give only the values obtained at speeds which result in the highest efficiency being obtained from the wheel. In practice c_e varies from 0.6 to 0.8 according to the type of runner. Then

$$q = A_1 \times c_e\sqrt{2gh}.$$

It may be of interest to note that as one proceeds from runners of Type I to Type IV of Fig. 34, page 43, one gets farther away from the impulse wheel in all respects. Not only are the resulting operating characteristics and conditions of service more unlike but the numerical factors are of increasing difference. Thus values of ϕ_e for the reaction turbine are larger than for the impulse turbine and they increase in the direction mentioned. The pressure p_1 is zero for the impulse turbine, but not for the reaction turbine. For the same head and setting, the value of p_1 will increase from Type I to Type IV. But if p_1 increases, V_1 must decrease. Hence high values of c_e accompany low values of ϕ_e and vice versa.

For the present we are assuming that values of ϕ_e and c_e are to be chosen according to the type of runner concerned.

84. Conditions for Maximum Efficiency.—To obtain the best efficiency the water must enter the runner without shock and leave with as little velocity as possible. In order to enter without shock the vane angle must agree with the angle β_1 determined by the velocity diagram and, in the case of the reaction turbine, the velocity v_1 as determined by the velocity diagram should be equal in magnitude to that determined by the rate of discharge and the runner area a_1. In order to leave with as little velocity as possible the angle α_2 may be made equal to 90°, as has been shown in Art. 64. In the early type of turbine as built by Francis such an

that with the high-speed type of runner the flow is approximately parallel to the axis.

A further reason for the use of $\alpha_2 = 90°$ as desirable for a high efficiency of the reaction turbine is that otherwise the water would enter the draft tube with a whirling motion which would increase the losses within the latter.

85. Determination of Speed for Maximum Efficiency.—A runner of rational design would be so proportioned that there would be no shock at entrance for the same speed at which the discharge velocity would be normal to the vane velocity. That is $\alpha_2 = 90°$ and $\beta'_1 = \beta_1$ at the same speed, where β'_1 is the angle of the runner vane and β_1 the angle of v_1 as determined by the vector diagram. The following equations therefore apply only to such a runner.

From the velocity diagrams we have, if $\beta'_1 = \beta_1$.

$$V_1 \sin \alpha_1 = v_1 \sin \beta'_1$$

$$V_1 \cos \alpha_1 = u_1 + v_1 \cos \beta'_1$$

Eliminating v_1 between these two equations we have

$$u_1 = \frac{\sin (\beta'_1 - \alpha_1)}{\sin \beta'_1} V_1 \tag{32}$$

as the relation between u_1 and V_1 when there is no abrupt change of velocity at entrance to the runner.

Since $\alpha_2 = 90°$ and hence $Vu_2 = 0$, we have from equation (19)

$$e_h h = \frac{u_1 V_{u1}}{g} = \frac{u_1 V_1 \cos \alpha_1}{g} \tag{33}$$

as the relation between u_1 and V_1 for which the discharge loss is a minimum.

Solving equations (32) and (33) simultaneously we have

$$u_1 = \sqrt{\frac{e_h 2gh \sin (\beta'_1 - \alpha_1)}{2 \sin \beta'_1 \cos \alpha_1}}$$

It must be borne in mind that the preceding equations apply only to a runner designed as stated. For any runner, whether of rational design or not, the value of ϕ necessary to make $\alpha_2 = 90°$ can be determined by involving more dimensions than the above, and such an expression will now be derived. It will be assumed also that this is the most efficient speed for any runner, though this may not be strictly true if the entrance loss is not zero at this speed.

If $\alpha_2 = 90°$, $V_{u2} = u_2 + v_2 \cos \beta_2 = 0$. Since $u_2 = xu_1$ and $v_2 = (A_1/a_2)\ V_1 = yV_1$ from the equation of continuity, we may write

$$x\ u_1 + y\ V_1 \cos \beta_2 = 0 \tag{36}$$

as the relation between u_1 and V_1 for which $\alpha_2 = 90°$. Solving this simultaneously with equation (33) we obtain

$$u_1 = \sqrt{\frac{e_h \cdot 2gh \cdot y \cos \beta_2}{-\ 2x \cos \alpha_1}}$$

$$V_1 = \sqrt{\frac{e_h \cdot 2gh \cdot x}{-\ 2y \cos \beta_2 \cos \alpha_1}}$$

From this it follows that

$$\phi_e = \sqrt{\frac{e_h y \cos \beta_2}{-\ 2x \cos \alpha_1}} \tag{37}$$

$$c_e = \sqrt{\frac{e_h \cdot x}{-\ 2y \cos \beta_2 \cos \alpha_1}} \tag{38}$$

From equation (33) we may write $u_1\ V_1 = e_h \cdot 2gh/2cos\ \alpha_1$ and from this it follows that

$$\phi_e c_e = e_h/2 \cos \alpha_1 \tag{39}$$

Values of e_h and α_1 change somewhat with different types of turbines but this shows that the factors ϕ_e and c_e vary approximately inversely, as stated in Art. 83.

The equations of this article are all based upon assumptions

$\frac{1}{g}(u_1 V_{u_1} - u_2 V_{u_2})$. In accordance with the usual method in hydraulics we may represent hydraulic friction loss in the runner by $k\ v_2^2/2g$, k being an experimental constant. If the turbine discharges into the air or directly into the tail race the discharge loss is $V_2^2/2g$. In addition there may be a shock loss at entrance to the runner. The term *shock* is commonly applied here but the phenomena are rather those of violent turbulence. This turbulent vortex motion causes a large internal friction or eddy loss.

Referring to Fig. 94, the value of v_1 and its direction are determined by the vectors u_1, and V_1. Since the wheel passages

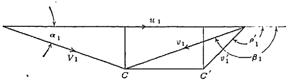


FIG. 94.

are filled in the reaction turbine, the relative velocity just after the water enters the runner is determined by the area a_1 and its direction by the angle of the wheel vanes at that point. If all loss is to be avoided, these values should agree with those determined by the vector diagram; but that is possible for only one value of u_1 for a given head. For any other condition the velocity v_1 at angle β_1 is forced to become v'_1 at angle β'_1. This causes a loss of head which will be assumed to be equal to $(CC')^2/2g$. Since the area of the stationary guide outlets normal to the radius should equal the area of the wheel passages at entrance normal to the radius, the normal component (*i.e.*, perpendicular to u_1) of v_1, should equal that of V_1. Therefore CC' is parallel to u_1 and its value is easily found to be

$$\text{shock loss} = \frac{(u_1 - k'V_1)^2}{2g}.$$

87. Relation between Speed and Discharge.—Equating the net head to the sum of all these items we have

$$h = k\frac{v_2^2}{2g} + \frac{V_2^2}{2g} + \frac{(u_1 - k'V_1)^2}{2g} + \frac{2(u_1V_{u_1} - u_2V_{u_2})}{2g}.$$

All velocities can be expressed in terms of u_1 and V_1 as follows:

$$u_2 = xu_1,\ v_2 = yV_1,\ V_{u1} = V_1 \cos \alpha_1,$$

$$V_{u2} = u_2 + v_2 \cos \beta_2 = xu_1 + yV_1 \cos \beta_2,$$

$$V_2^2 = u_2^2 + v_2^2 + 2u_2v_2 \cos \beta_2 = x^2u_1^2 + y^2V_1^2 + 2xyu_1V_1 \cos \beta_2.$$

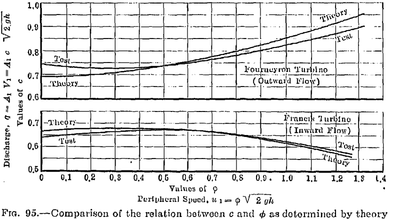


FIG. 95.—Comparison of the relation between c and ϕ as determined by theory and by test.

Making these substitutions and reducing we obtain,

$$[(1 + k)y^2 + k'^2]V_1^2 + 2(\cos \alpha_1 - k')V_1u_1 + (1 - x^2)u_1^2 = 2gh.$$

From this equation V_1 may be computed for any value of wheel speed, u_1. It is customary to express the wheel speed as $u_1 = \phi\sqrt{2gh}$, and we may also say $V_1 = c\sqrt{2gh}$. The use of these

One turbine was an outward flow turbine and the other was a radial inward flow turbine. Considering the imperfections and limitations of the theory, the agreement is remarkably close.

If the turbine discharges into an efficient draft tube the discharge loss may be represented by $mV_2^2/2g$, where m is a factor less than unity. If there were no internal friction and eddy looses within the tube, the value of m would depend only upon the areas of ends of the tube and would be equal to $(A_2/A_3)^2$. Actually m is greater than this due to hydraulic friction losses. And as the speed of the turbine departs from the normal value, it is probable that m increases still more and approaches unity. Introducing the discharge loss as $mV_2^2/2g$ in the equation at the beginning of this article, we obtain as a substitute for equation (40),

$$[(m + k)y^2 + k'^2]c^2 + 2[\cos \alpha_1 - k' - (1 - m)xy \cos \beta_2]c\phi - [(2 - m)x^2 - 1]\phi^2 = 1 \quad (41)$$

It will be found that this equation will give slightly higher values of c than equation (40), which is to be expected. Thus the use of a diverging draft tube increases the power of the turbine not only by increasing its efficiency but also by increasing the quantity of water it can discharge.

If desired, equation (36), when put in terms of ϕ and c, can be solved simultaneously with equations (40) or (41), thus giving a third method of computing the value of ϕ_e. Also it is possible to derive a general equation for the efficiency of a reaction turbine and by calculus find the value of ϕ for which the efficiency is a maximum. However the resulting equation is somewhat lengthy and, because it is of no practical value, will not be given here. Values of ϕ determined by it will usually not differ much from those determined by the simpler approximate method of assuming that $\alpha_2 = 90°$.

88. Torque, Power and Efficiency.—General equations for torque, power, and efficiency were derived in Chapter VII and the application of these illustrated by a numerical case in Art. 63.

terms of known factors and dimensions. Thus, to illustrate, the hydraulic efficiency is in general

$$e_h = h''/h = (u_1 V_1 \cos \alpha_1 - u_2 V_2 \cos \alpha_2)/gh.$$

For the reaction turbine in particular $u_2 = xu_1$ and $V_2 \cos \alpha_2 = u_2 + v_2 \cos \beta_2 = xu_1 + yV_1 \cos \beta_2$. Substituting in the above we obtain $e_h = [(\cos \alpha_1 - xy \cos \beta_2)\, V_1 u_1 - x^2 u_1^2]/gh$. From this it follows that

$$e_h = 2(\cos \alpha_1 - xy \cos \beta_2)\, c\phi - 2x^2\phi^2 \qquad (42)$$

A numerical result in a given case can be computed either by substituting the known quantities in the above equation or by computing the separate items of the general equation. The latter usually involves no more labor.

Equation (42) is of interest because it involves no arbitrary factors of loss. Thus if the relation between speed and discharge is known, as by experiment, the hydraulic efficiency can be computed, provided the proper wheel dimensions are known. Actually it is so difficult, as will be explained later, to determine the proper values of the runner dimensions, that the numerical accuracy of the result is doubtful. The hydraulic efficiency can probably be estimated more accurately than the separate factors in these equations. The euqation is of very practical value however in showing that the hydraulic efficiency is independent of the head under which the turbine is run.

Equation (42) is perfectly general for any reaction turbine and is not restricted to the maximum efficiency. The value of the maximum efficiency will be obtained by using the values of ϕ_e and c_e in the equation. Of course, since V_u is assumed to be zero for this case the value of the maximum efficiency can be computed much more directly than by the use of the above.

89. Variable Speed—Constant Gate Opening.—Since c varies with the speed the input for a fixed gate opening will not be constant for all speeds as it is in the case of the impulse turbine. The variation of the losses at full gate with the speed ranging from zero

that the speed at which the efficiency is a maximum will be slightly different from the speed for which the power is the greatest.

The Francis turbine for which the curves in Fig. 95 and Fig. 96 were constructed had the following dimensions:

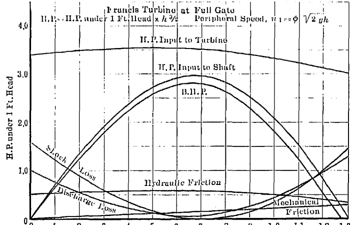


FIG. 96.—Losses at full gate and variable speed.

$$\alpha_1 = 13^\circ,\ \beta'_1 = 115^\circ,\ \beta_2 = 165^\circ,\ A_1 = 5.87 \text{ sq. ft.},\ a_2 = 6.83 \text{ sq. ft.},\ r_1 = 4.67 \text{ ft.},\ r_2 = 3.99 \text{ ft.}$$

From this data $x = 0.855$, $y = 0.860$, $k' = 1.08$, and $k = 0.5$ (assumed). Attention is called to the fact that the horse-power output was determined by an actual brake test while the horse-power input to shaft was computed from the theory given in the preceding article. The two differ by the amount of power consumed in bearing friction and other mechanical losses.

90. Constant Speed—Variable Input.—The relation between input and output and the segregation of losses for a cylinder gate turbine at constant speed is shown in Fig. 97. In the four tests

water. When the turbine is running at the normal speed but with the gate partially closed there is a shock loss of a slightly different nature. A partial closure of the gates increases the value of V_1 and the angle α_1 may be affected somewhat. However q will be reduced while α_1 remains the same and thus the

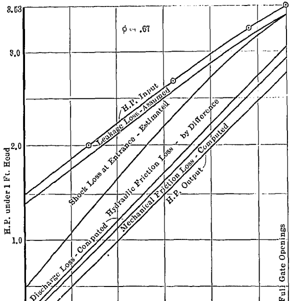


FIG. 97.—Losses for cylinder gate Francis turbine at constant speed.

velocity v_1 must be suddenly reduced to v'_1. The loss of head due to this may be roughly represented by $(v_1 - v'_1)^2/2g$. While this expression may not give the exact value of the loss, yet it must be true that it will be of the nature shown by the curves.

This loss is known to be less in the case of the swing gate

large shock losses and hence reduce the input curve to a line more nearly parallel to the output line in Fig. 97, and thus improve the part load efficiency of the turbine. Also the cylinder gate turbine gives its best efficiency when the gate is completely raised and the

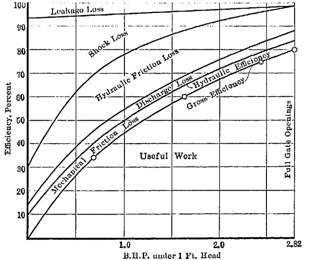


FIG. 98.—Cylinder gate turbine at constant speed.

power output has its greatest value. But the wicket gate turbine usually develops its best efficiency before the gates are fully open. There is thus left some overload capacity.

91. Runner Discharge Conditions.—The following theory, though open to certain objections, serves to explain the observed phenomena at the discharge from a turbine runner. A low specific speed type of runner, such as Type I in Fig. 34, page 43, will have stream lines through it which differ but little from one another, while a high specific speed type such as Fig. 99 will have stream lines which differ considerably. Thus in Fig. 99 stream line (*a*) next to the crown will have smaller radii at both

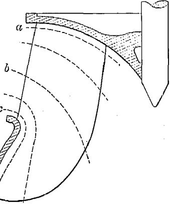


FIG. 99.

also in the draft tube, does not exist within the runner. But, considering the velocity component in the plane of the paper only, and considering the rotation about the centers of curvature of the lines drawn (and not about the axis of the runner), the equations of Art. 66 may be applied.

Since $e_h = h''/h = (u_1V_{u1} - u_2V_{u2})/gh$, we may write

$$u_2V_{u2} = u_1V_{u1} - gh'' \quad (43)$$

not only for the turbine as a whole, but for each individual stream line. It has been stated that usually a runner is so designed that $\alpha_2 = 90°$. With some runners observation shows that there is a slight whirl of the water across the entire draft tube at the point of maximum efficiency, but this might be expected, since the assumption that V_{u2} should equal zero is a mere approximation. With the low specific speed turbine it is possible to have $\alpha_2 = 90°$ for all stream lines at some speed which may or may not be exactly the most efficient, but it is difficult to do this with the high specific speed runner and still satisfy the equation above. Thus suppose the discharge is normal to u_2 for stream line (*b*) in Fig. 99. Then for this stream line $u_1V_{u1} = gh''$. Considering line (*c*) both u_1 and V_{u1} are larger for the reasons stated in the first paragraph. If V_{u2} is to be zero here also, h'' must be larger. But the conditions here are not favorable to as high an efficiency as along line (*b*), because of the proximity to the boundary (which

Hence by no proportioning can the water be compelled to flow as desired. Since $u_1 V_{u1}$ is larger and h'' is smaller than for line (*b*) it follows that the right hand member of the equation must be positive and hence there must be some whirl at the point of discharge in the direction of rotation of the runner. In similar fashion there may be a negative whirl at the point of discharge from line (*a*), but since u_1, Vu_1, and h'' all decrease for this line, as compared with (*b*), it is possible that there may be little or no whirl here. All this reasoning has been verified by experimental observations.[1] This whirl of the water near the band decreases the efficiency of the draft tubes as constructed in the past and points the reason for the development of a new type of tube if turbines of higher specific speed are to be used. And unless more effective draft tubes are used, this shows that this factor tends to reduce the efficiency of the runner as the specific speed increases.

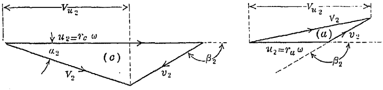


FIG. 100.

For the higher the specific speed the greater the variation in r_2 for stream lines (*a*) and (*c*), and this has been shown to be undesirable.

At part gate on any turbine the efficiency and hence h'' are known to be less than on full load, the latter being taken as the load for which the efficiency is a maximum. And if wicket gates are used the angle α_1 is less than at full load so that $V_1 \cos \alpha_1$ would appear to be higher. Hence if the right hand side of equation (43) is equal to zero at full load, it would have a relatively large positive value for a partial opening of the turbine gates. Thus V_{u2} would be positive, which agrees with the vector dia-

radii of the discharge edge of a runner, and this latter is characteristic of the profile of the high specific speed runner. Hence this theory presents a reason why the part gate efficiency of a reaction turbine must be less as the specific speed increases.

The smaller the rate of discharge for a given head the less the value of v_2 at any point on the outflow edge and hence the less the wheel speed necessary to make $\alpha_2 = 90°$ at this point. Thus with any turbine the speed for which the efficiency is a maximum decreases as the gate opening decreases.

92. Limitations of Theory.—The defects of this theory or any theory are as follows: In order to apply mathematics in any simple way it is necessary to idealize the conditions of flow by assuming that all the particles of water at any section move in the same direction and with the same velocity. Such is very far from being the case so what we use in our equations is the average direction and the average velocity of all the particles of water. That in itself could easily cause a discrepancy between our theory and the fact, because the theory is incomplete.

But even to determine accurately these average values that are used in the equations is a matter involving some difficulty. Thus, though the direction of the streams leaving the runner is influenced by the vane angle at that point, it cannot be said that the angle β_2 is exactly equal to the vane angle at exit. In fact the author has roughly proved by study of a test where some special readings were observed that the two may differ by from 5 to 10 degrees, and that β_2 varied regularly for different values of ϕ. The same thing may be said about the area a_2. Some recent experiments in Germany[1] have shown that there may be a certain amount of contraction of the streams and that this contraction varies for different speeds. Thus the true value of a_2 may be slightly less than the area of the wheel passages. These observations concerning β_2 and a_2 apply equally well to other angles and areas.

[1] *Zeitschrift des Vereines deutscher Ingenieure*, May 13, 191

is doubtless true that k is not strictly constant here. If it were known just how k and the dimensions used varied with the speed, the theoretical curve could be made to more nearly coincide with the actual curve. In addition the expression for shock loss is only an approximation. But even as it is the discrepancy is not serious.

By the use of the proper average dimensions the equations given may be successfully applied to a radial flow turbine. For the mixed flow turbine they will apply approximately. The

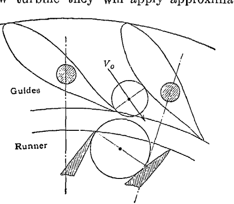


Fig. 101.

reasons for this are that with the mixed flow turbine the radius r_2 varies through such a wide range of value that it is difficult to fix a proper mean value; likewise the vane angle at exit and also the area varies so radically that a mean value can scarcely be obtained with any accuracy. Even if these mean values could be obtained the theory would still be imperfect, for the reason stated in the first paragraph of this article.

The value of the angle α_1 may be taken as that of the angle shown in Fig. 101, though it may be seen that this is a mean for the various stream lines. The velocity of the water through the guide vanes may be denoted by V_0 but, since the space between

by applying the laws of hydrodynamics, such as equations (40), (41), and (42) for example, it is desirable to select the point (1) such that the most suitable mean values for use in the equations will be obtained.[1] Such a point is often said to be the center of the circle shown at entrance to the runner of Fig. 101. The diameter of this circle is the shortest line that can be drawn from the tip of one vane to the next vane.

A similar procedure should be followed for the point of discharge if the turbine were a pure radial flow turbine with all points on the outflow edge at the same radius. For the mixed flow type of turbine it can be proven that the discharge may be considered as concentrated at the center of gravity of the outflow area.

Because of the difficulties of applying the theory in a definite case numerical results are of doubtful accuracy. But the theory has other uses. Thus the theory shows why certain factors and dimensions must vary with the specific speed of the runner. It shows that the rate of discharge cannot be constant for a given runner at different speeds under a constant head and gate opening. It shows why certain conditions are desirable for efficiency and how the proper speed may be approximately computed. It explains the losses within a turbine and shows why certain characteristics vary as they do. It serves to give the reasons why there are fundamental differences in the operating characteristics of turbines of different types. In other words it will in general furnish the reason for any result found in practice. And beyond explaining these characteristics, it indicates the effect of any change in any direction.

93. Effect of y.—The ratio of A_1/a_2 is expressed by y. If y is small enough the turbine will be an impulse turbine, the value of ϕ giving the best speed will be about 0.45, $p_1/w = 0$, and c will equal 1.00 if the slight loss in the nozzle is neglected. (Ac-

[1] This procedure was not followed however in dealing with the Francis turbine for which the curves in this chapter were drawn. But this turbine

It is thus seen that the design of a reaction turbine can be varied so as to secure quite a range of results.

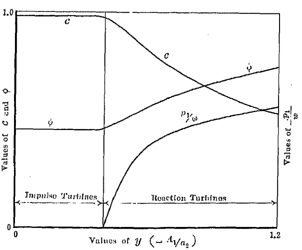


FIG. 102.

94. QUESTIONS AND PROBLEMS

1. Given the diameter D and the height B of a turbine runner, how can one approximately compute the speed and power for any head?

2. Why does the rate of discharge from a turbine runner vary with the speed under a fixed head? Why is the velocity of the water entering the runner less than $\sqrt{2gh}$?

3. What are the conditions necessary for high efficiency of a reaction turbine? What effect does the draft tube have upon this also?

4. In what two ways may ϕ_e be computed? What are the fundamental differences involved in these methods? Should the numerical results differ?

5. What are the various losses of the turbine and how may they be expressed? What is the effect of the draft tube in this?

6. How may a general equation between speed, discharge, and head be derived?

7. How may a general equation for the hydraulic efficiency of a reaction turbine be derived? What does it indicate?

8. From the curves, what are the differences between the variations in

value of the theory?

12. What effect does the change in the ratio of the area through the guide vanes to that at outflow from the runner have upon the values of ϕ, c, and p_1/w?

13. A turbine runner 36 in. in diameter and 12 in. high at entrance will run at what probable r.p.m. and develop what power under a head of 60 ft.? (Assume value of α_1.) *Ans.* $N = 317$, B.h.p $= 890$.

14. In problem (13) suppose the intake to the runner is at a height of 15 ft. above the tail-water level. What is the probable value of the pressure head at this point? *Ans.* 29 ft.

15. The dimensions of the original Francis runner were $\alpha_1 = 13°$, $\beta'_1 = 115°$, $\beta_2' = 165°$, $A_1 = 5.87$ sq. ft., $a_2 = 6.83$ sq. ft., $r_1 = 4.67$ ft., and $r_2 = 3.99$ ft. Compute the values of ϕ_e and c_e by the first method given, assuming $e_h = 0.83$. Do these answers give shockless entrance? Do they give $\alpha_2 = 90°$? What dimensions could be changed to make both of these conditions be fulfilled at the speed computed? *Ans.* $\phi_e = 0.678$, $c_e = 0.628$.

16. Compute the values of ϕ_e and c_e for the Francis turbine in the preceding problem by the second method given? Do these answers give shockless entrance? Do they give $\alpha_2 = 90°$? What dimensions could be changed so as to fulfill both these conditions at this speed?

Ans. $\phi_e = 0.643$, $c_e = 0.663$.

17. Francis noted that his runner was not quite properly designed and that there was some shock loss at entrance when running at the most efficient speed. By test the actual value of ϕ_e was found to be 0.67. Compute the corresponding value of c_e and compare with the curve in Fig. 95. Assume $k = 0.5$. *Ans.* $c_e = 0.655$.

18. Compute the hydraulic efficiency of the Francis turbine of problem (15) using the values of ϕ and c given in problem (17) and compare with value given by curve in Fig. 98. *Ans.* 0.825.

19. If this turbine discharges into a draft tube of such dimensions that m may be assumed equal to 0.3, compute the value of c for a value of ϕ_e equal to 0.675. Compute the hydraulic efficiency. The value of ϕ_e has been increased slightly here because of the presumption that the draft tube will increase the efficiency of the turbine. Compare with problems (17) and (18).

Ans. $c_e = 0.66$, $e_h = 0.833$.

20. What is the percentage value of the discharge loss from the Francis turbine of problem (15), assuming $\alpha_2 = 90°$ and $c_e = 0.66$? For this particular turbine, what is the possible gain in efficiency due to using a draft tube which would reduce the velocity to zero without loss of energy? (Note. $V_2 \sin \alpha_2 = V_2$ for 90° and $V_2 \sin \alpha_2 = v_2 \sin \beta_2 = yc\sqrt{2gh} \sin \beta_2$.)

Ans. 2.15 per cent.

21. If the turbine in problem (17) is used under a head of 30 ft., find the

Hydraulic efficiency and power.
$\phi = 0.864$, $c = 0.633$ by (40), $q = 126.5$, $e_h = 0.755$, 195 h.p.

TURBINE TESTING

95. Importance.—Testing is necessary to accompany theory in order that the latter may be perfected until it becomes reliable enough to be useful. Unless the theory agrees with the facts it is not true theory but only an incorrect hypothesis. Only by means of theory and testing working hand in hand can improvements in design be readily brought about. Thus the ease of testing is a measure of the rate of development of any machine.

Again, if we are to thoroughly understand turbines, it will be necessary to make a thorough study of test data in order to appreciate the differences between different types. Unfortunately there is a scarcity of good and thorough test results.

The only public testing flume in the United States is the one at Holyoke, Mass. Nearly 3000 runners have been tested there and it has been an important factor in the development of modern turbines. The maximum head obtainable there is about 17 ft., also it is scarcely possible to test runners above 42 in. in diameter because of the limitations imposed by the depth of the flume.

An acceptance test should always be made when a turbine is purchased if it is possible to do so. Otherwise the purchaser will have no assurance that the specifications have been fulfilled. Thus a case may be cited where the power and efficiency of a tangential water wheel were both below that guaranteed as can be seen by the following:

	Efficiency	Normal h.p.	Maximum h.p.
Guarantee....................	0.800	3500	5225
Test..........................	0.720	2300	3500

In this table the normal horse-power means the power at which the maximum efficiency is obtained, any excess power over that being regarded as an overload. The actual efficiency is 8 per

for a reaction turbine is shown in Fig. 103. The efficiency secured was higher than that guaranteed, but it was also attained at a much higher horse-power. If the turbine were then run on the load specified it would be operating on part gate all the time and at a correspondingly low efficiency. This is a common failing in "cut and try" practice. A turbine of excess capacity is

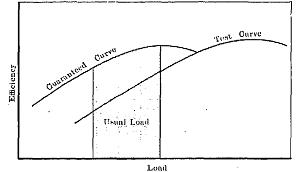


Fig. 103.

provided; it never lies down under any load put upon it and the owner is satisfied. Quite frequently also a turbine which must run at a certain speed is really adapted for a far different speed. Thus under the given conditions its efficiency may be very low, when the runner might really be excellent if operated at its proper speed. A test would show up these defects, otherwise they may remain unknown.

Another reason for making tests would be to determine the condition of the turbine after length of service. The effect of seven years' continuous operation upon a certain tangential water wheel is seen in Fig. 104. This drop in efficiency is due to roughening of the buckets, to wear of the nozzle, and to the fact that end play of the shaft together with the worn nozzle caused the jet to strike upon one side of the buckets rather than fairly in the center. It might be noted however that a 7-ft. wheel of the

demanded. But where the water must be ... some cases, or where vast storage reservoirs are constructed at considerable expense it is desirable that water be used with the utmost economy.

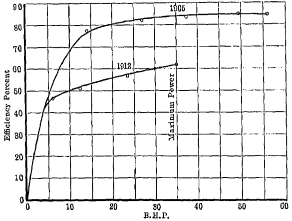


FIG. 104.—Effects of service upon a 42″ tangential water wheel.

96. Purpose of Test.— The nature of the test will depend upon the purpose for which it is made. In a general way there are four purposes as follows:

1. *To Find Results for Particular Specified Conditions.* This will usually be an acceptance test to see if certain guarantees have been fulfilled. The guarantee will usually specify certain values of efficiency obtained at certain loads at a fixed speed under a given head. Occasionally several values of the head will be specified.

2. *To Find Best Conditions of Operation.* Such a test will cover a limited range of speed, load, and head; all of them, however, being in the neighborhood of the maximum efficiency point. A test of this nature will show what a given turbine is best fitted for.

etter and could also be used to verify the theory.

4. *To Investigate Losses.*—This test would be similar to the receding except that a number of secondary readings of velocies, pressures, etc., at various points might be taken. Such a st will be of interest chiefly to the designer.

97. Measurement of Head.—The head should be measured as ose to the wheel as possible in order to eliminate pipe-line losses. he head to be used should be as specified in either equation (7) (8) or (9) of Art. 55, according to circumstances. The pressure ay be read by means of a pressure gage if it is high enough. or lower heads, a mercury column or a water column will give ore accurate results. Care should be taken in making connecons for the pressure reading so that the true pressure may be btained. The reading of any piezometer tube will be correct ly when the tube leaves at right angles to the direction of flow d when its orifice is flush with the walls of the pipe. No tube rojecting within the pipe will give a true pressure reading, even ough it be normal to the direction of flow.[1]

98. Measurement of Water.—The chief difficulty in turbine sting is the measurement of the water used. In some commeral plants the circumstances are such that it is scarcely possible measure the water at all and in others the expense is prohibive. The necessity of cheap and accurate means of determining e amount of water discharged is imperative.

The standard method of measurement is by means of a weir. or large discharges, however, the expense of constructing a itable weir channel may be excessive, and, in case the turbine scharges directly into a river, it may be almost impossible to nstruct it. In the case of a turbine operating under a low head e increase in the tail-water level caused by the weir may cause serious decrease in head below that normally obtained. This ould make the test of little value. However, where it is feasible, e use of a weir is a very satisfactory method and should be proded for when the plant is constructed. It should be remem-

vel, but it will not be as great as the weir. At present enough perimental data had not been gathered to make this method plicable in general, but perhaps in the future it may be used th fair success.

Either in the tail race or in the head race a Pitot tube, current eter, or floats may be used. These methods involve no disrbance of the head under which the turbine ordinarily operates, t they do require a suitable channel in which the observations n be taken. These instruments should be in the hands of a illed observer who understands the sources of error attendant on their use.[2]

The Pitot tube consists of a tube with an orifice facing the rrent. The impact of the stream against this orifice produces certain pressure which is proportional to the square of the locity. If h is this reading in feet of water and K and experiental constant, then

$$V = K\sqrt{2gh}.$$

nce it would be very difficult to determine accurately the height the column of water in a tube above the level of the stream is customary to use two tubes and read the difference between e two. For convenience in reading, the instrument is made so at valves may be closed and the device lifted out of the water thout changing the levels of the columns, or sometimes both lumns may be drawn up to a convenient place. The orifice r this second tube is usually in a plane parallel to the direction flow and will thus give a lower reading than the other. It does t, however, give the value of the pressure at that point, as ated in Art. 97. For low velocities it is desirable to magnify is difference in the two readings and for that purpose the orifice the second tube may be directed down stream. Its reading ll then be less than for the one parallel to the direction of

[1] See "Weir Experiments, Coefficients, and Formulas," by R. E. Horton, S. G. S. Water Supply and Irrigation Paper No. 150, Revised, No. 200.
[2] See Hoyt and Grover, "River Discharge."

The Pitot tube may also be used in a pressure pipe. Since the reading of the impact tube alone will be the sum of the pressure head plus the velocity head it will be necessary to use two tubes in the same manner as in the case of the open channel. The value of h will be the difference between these two readings, and the value of K must be determined experimentally. If, however, only one orifice is used and the pressure is determined by a piezometer tube with its orifice lying flush with the walls of the pipe the difference between these two readings may be considered equal to the velocity head, that means the value of $K = 1.0$.

For the tangential water wheel the Pitot tube may also be used to determine the jet velocity. In such a case only the impact tube is required. While it is well to determine the value of K experimentally, yet if the tube is properly constructed it may be taken as 1.0. A check on this may be obtained as follows: It is probably true that the maximum velocity obtained at any point in the jet is the ideal velocity. The latter can be computed from the head back of the nozzle and the value of K should be such as to make the two agree. Either in the pipe or the jet it is desirable to take a velocity traverse across each of two diameters at right angles to each other. In computing the average velocity it is necessary to weight each of these readings in proportion to the area affected by them.[1]

Chemical methods are often of value. If the pipe line is sufficiently long a highly colored stain may be added to the water at intake and the time noted that it takes the color to appear in the tail race. From this and the pipe dimensions the rate of discharge can be computed. A second chemical method is to inject a salt solution into the water of known concentration and

[1] See "Application of Pitot Tube to Testing of Impulse Water Wheels," by Prof. W. R. Eckart, Jr., *Institution of Mechanical Engineers*, Jan. 7, 1910. Also printed in *Engineering* (London), Jan. 14, 21, 1910.

Engineering News, Vol. LIV, Dec. 21, 1905, p. 660. See also *Zeitschrift des ver. deut. Ing.*, Mar. 22, 29, and Apr. 5, 1913. For useful information regarding all the methods of measurement given here see Hughes and Safford.

the most satisfactory method is to use some ... brake or absorption dynamometer.[2]

The use of a simple brake is restricted to comparatively small powers. For large powers it becomes rather expensive and difficult. A good absorption dynamometer may be used satisfactorily for fairly large powers but the drawback is one of initial expense. In many cases also where turbines are direct connected to electric generators it may be impossible to attach a brake of any kind.

In such cases it is necessary to supply an electrical load for the generator and determine the generator efficiency. However, this method of testing involves a number of instrument readings which may be more or less in error and a rather complicated process of computation. Nevertheless it can be done with very satisfactory results. One drawback about it is that the speed cannot be varied through the same range of values as in the brake test. The output of the generator may be absorbed by a water rheostat which will furnish an absolutely constant load. If it is a direct-current machine this rheostat may simply consist of a number of feet of iron wire wound on a frame and immersed in water to keep it cool. This water should be running water or a large pond so that its temperature may not change. The current is shorted through this coil; the load is varied by changing the length of wire in use. For a three-phase alternator the rheostat may consist of three iron pipes at the vertices of an equilateral triangle with a terminal connected to each. The load is varied by changing the depth of immersion of the pipes in water.[3]

A good method recently employed in a hydro-electric plant where there are two or more similar units is to let one alternator drive the other as a synchronous motor. The second rotates the impulse wheel or reaction turbine in the reverse direction.

[1] B. F. Groat, "Chemi-hydrometry and precise Turbine Testing," *Trans. A. S. C. E.*, Vol. LXXX, p. 951 (1915).

[2] C. M. Allen, "Testing of Water Wheels after Installation," *Journal A. S. M. E.*, April 1910.

a smooth curve should be drawn in all cases. Also if any readings should follow a law which is any approach to a straight line it is better to work from values given by that line rather than from the experimental values themselves. Thus if a turbine be tested at constant gate opening and at all speeds, the curve showing the relation between speed and efficiency may be drawn at once from the experimenal data. However, a more accurate curve can be constructed by noting that the relation between speed and brake reading is a fairly straight line. See Fig. 86. This is not a straight line but the curvature is not very marked so that it may be drawn readily and accurately. Values given by this curve may then be used for constructing the efficiency curve. Again, when a turbine is tested at constant speed, it should be noted that the relation between input and output is not a straight line absolutely, but it is approximately so. If any point falls decidedly off from a straight line it is probably in error. From the line giving the relation between input and output the efficiency curve may be constructed.

In computing the true power in a jet it might also be noted that it is not that given by using the square of the average velocity but something 1 or 2 per cent. higher than that. The reason is that the velocity throughout the jet varies and the summation of the kinetic energy of all the particles is not that obtained by using the average velocity.[1]

101. Determination of Mechanical Losses.—With the tangential water wheel the mechanical losses will consist of bearing friction and windage. With the reaction turbine they will consist of the bearing friction and the disk friction due to the drag of the runner through the water in the clearance spaces. There are several ways of determining this but the retardation method is probably as satisfactory as any. The turbine is brought up to as high a speed as is possible or desirable and the power shut off. As the machine slows down readings of instan-

[1] L. M. Hoskins, "Hydraulics," p. 119.

readings will then enable us to find the average speed corresponding to the middle of this time interval.

The power lost at any speed is equal to a constant times the subnormal to the curve at that speed. If L equals the power lost then

$$L = K \times BD.$$

For the proof of this proposition see Appendix A.

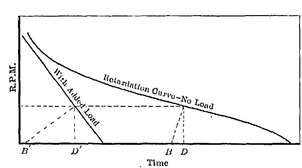


Fig. 105.—Retardation curves.

To determine the value of the constant K a second run is necessary with a definite added load. This load, which may be small, may be obtained by closing the armature circuit on a resistance if a generator is used in the test or by applying a known torque if a Prony brake is used. With the first method a watt-meter should be used and the load kept constant for a limited range of speed, with the second method the torque should be kept constant for a limited range of speed. If this known added load be denoted by M we then have

$$L + M = K \times B'D'.$$

Since L is the only unknown quantity except K it may be eliminated from these two equations and we have

$$K = \frac{M}{}$$

1. What is the value of testing a turbine upon installation? What is the alue of testing one that has been in operation for some time? What is ne value of a Holyoke test to the purchaser of another runner but of similar ype? Is it cheaper to increase the power output of a plant by additional onstruction or by improving the efficiency?

2. What are the various purposes for which turbine tests may be conucted? What conditions would be varied for each of these and what kept onstant?

3. What methods of measuring the rate of discharge are usually employed n turbine tests?

4. In what ways may the power output of a waterwheel be absorbed and easured?

5. A case is reported where tests conducted at an expense of \$5000 resulted n changes which improved the efficiency of the turbines 1 per cent. If the apacity of the plant is 100,000 h.p. and 1 h.p. is worth \$100, what would e the value of the gain in efficiency, assuming the changes cost \$20,000?

6. In the Cedars Rapids turbines the area of the water passages at enrance to the casing = 1080 sq. ft. per unit, elevation of section above tail ater = 10 ft., and pressure head at this point = 20 ft. The area of the outh of the draft tube = 1050 sq. ft. The test showed the power output o be 10,800 h.p. with a rate of discharge of 3450 cu. ft. per second. Callate two values of the efficiency, using two values of the head.

7. In the test of a reaction turbine the water flowing over the weir in the ail race was found to be 39.8 cu. ft. per second. The leakage into the tail ace was found to be 1 cu. ft. per second. The elevation of the center line f the shaft above the surface of the tail water was 12.67 ft. The diameter f the turbine intake was 30 in. and the pressure at this section was measred by a mercury U tube. The readings in the two sides of the mercury tube were 10.556 ft. and 0.900 ft., the zero of the scale being at a level .82 ft. below that of the center line of the turbine shaft. The generator utput was 391.8 kw., friction and windage 13.8 kw., iron loss 2.0 kw., and rmature loss 4.4 kw. The specific gravity of the mercury used was 13.57. ind: Input to turbine, output of turbine (generator being excited from nother unit), efficiency of turbine, efficiency of generator, efficiency of set.

Ans. $h = 141.80$ ft., 625 w.h.p., 550 b.h.p., 0.880, 0.951, 0.837.

103. Head.—The theory that has been presented has made it clear that the speed and power of any turbine depends upon the head under which it is operated. The peripheral speed of any runner may be expressed as $u_1 = \phi\sqrt{2gh}$. It has also been shown that for the best efficiency ϕ must have a certain value depending upon the design of the turbine. It is thus apparent that the best speed of a given turbine varies as the square root of the head.

The discharge through any orifice varies as the square root of the head, and a turbine is only a special form of discharge orifice. Since $V_1 = c\sqrt{2gh}$, and since a definite value of c goes with the best value of ϕ as given above, it follows that the rate of discharge of a given turbine varies as the square root of the head.

Since the energy of each unit volume of water varies as the head, and since the amount of water discharged per unit time varies as the square root of the head it must then be true that the power input varies as the three halves power of the head.

In reality the rate of discharge through any orifice is not strictly proportional to the square root of the head, that is, the coefficient of discharge is not strictly a constant but varies slightly with the head. However, the variation in the coefficient is small and inappreciable except for very large differences in the head. Therefore the above statements are accurate enough for most practical purposes.

The theory has also shown that the losses of head in any turbine vary as the squares of the various velocities concerned. This rests upon the assumption that the coefficient of loss k is constant for all values of h as long as ϕ remains constant. That is probably not true, but may be assumed as true for all practical purposes. Since these velocities vary as the square root of the head their squares will vary as the first power of the head. The amount of water varies as the square root of the head and, since

unchanged. The mechanical losses really follow different laws at different speeds, as can be seen in Fig. 106. The factors which influence this are rather complicated and it does not seem possible to lay down any rule to express mechanical losses as a function of the speed. It is probably true, however, that these losses increase faster than the first power of the speed but not much faster than the square of the speed. Since the speed varies as

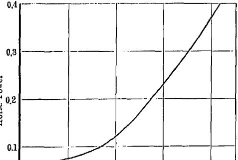


FIG. 106.—Friction and windage of a 24" tangential water wheel.

the square root of the head it is seen that if the friction losses vary at the same rate as the hydraulic losses they must increase as the cube of the speed. As they do not do so, it is apparent that the gross efficiency will be higher the higher the head under which the turbine operates. The change in the gross efficiency with a change in head is most apparent when the latter is very low. As the head increases the mechanical losses become of smaller percentage value and the gross efficiency tends to approach the hydraulic efficiency, which is constant, as a limit. Thus there is little variation in efficiency unless the head is very low.

The mechanical losses are really comparatively small being

essary to test a turbine under a certain head which is different from the head under which it is to be run. The question may then arise as to how far the test results can be applied to the new head. As long as the two heads are not radically different we may state that they will apply directly. If there is a large difference in head we may expect that the efficiency under the higher head may be one or two or more per cent. higher. This is borne out by some tests made by F. G. Switzer and the author where the head was varied from 30 ft. to 175 ft. and later from 9 ft. to 305 ft.

It is customary to state the performance of a turbine under one foot head. Then by means of the above relations we may easily tell what it will do under any head. If the suffix (1) denotes a value for one foot head we may then write,

$$N = N_1 \sqrt{h} \tag{44}$$

$$q = q_1 \sqrt{h} \tag{45}$$

$$h.p. = h.p._1 \, h^{3/2 *} \tag{46}$$

It must be noted that these simple laws of proportion may be applied only when the speed varies with the head in such a way as to keep ϕ constant. The value of ϕ need not necessarily be that for the highest efficiency. But if the speed does not change or if it varies in some other way so that ϕ is different, the results under the new head cannot be computed save by complex equations, such as those of Arts. 87 and 88, or by the use of test curves such as those of Figs. 95 and 96.

104. Diameter of Runner.—When a certain type of runner has been perfected a whole line of stock runners of that type may then be built with diameters ranging from 10 to 70 in. or more. All of these runners will be homologous in design, that is they will have the same angles and the same values of the ratios x, y, and B/D. Each runner will simply be an enlargement or reduction of another. They will then have the same characteristics, that is, the same values of ϕ_e and c_e, and will therefore follow certain laws of proportion.

of such a series, the discharge will be proportional to the area A_1. But if the runners are strictly homologous the area A_1 will be proportional to the square of the diameter. It will therefore be true that the discharge of any turbine of the series will be proportional to the square of the diameter.

Since the power is directly related to the discharge it also follows that the power of the turbine is proportional to the square of the diameter.

These relations are of practical value because if the speed, discharge, and power of any runner is known by accurate test, predictions may then be made regarding the performance of any other runner of the series. These laws may not hold absolutely in all cases because the series may not be strictly homologous, that is the larger runners may differ slightly from the smaller ones. Also it will no doubt be true that the efficiency of the larger runners will be somewhat higher than that of the smaller ones. It may also be found that careful tests of two runners made from the same patterns will not give exactly the same results due to difference in finish or other imperceptible matters. Despite these factors, however, the relations stated are true enough to be used for most purposes.

105. Commercial Constants.—For a given turbine the maximum efficiency will be obtained only for a certain value of ϕ. All tables in catalogs of manufacturers as well as all values given in this chapter are based upon the assumption that the speed will be such as to secure this value of ϕ. Substituting values of N and D for u_1 in the expression $u_1 = \phi\sqrt{2gh}$, we obtain

$$N = \frac{1840\phi\sqrt{h}}{D} \tag{47}$$

where D is the diameter of the runner in inches. From this may also be written

$$\phi = 0.000543 \frac{DN}{\sqrt{h}} \tag{48}$$

Since ϕ_e is constant for any series of runners of homologous

$$\frac{DN}{\sqrt{h}} = 790 \text{ to } 870.$$

For the reaction turbine:

$$\phi_e = 0.55 \text{ to } 0.90.$$

$$\frac{DN}{\sqrt{h}} = 1050 \text{ to } 1600.$$

If values outside these limits are met with it is because the speed is not the best or because the nominal value of D is not the true value.

106. Diameter and Discharge. Since, for any fixed gate opening and a constant value of ϕ, the rate of discharge of any runner is proportional to the square of its diameter and to the square root of the head, we may write

$$q = K_1 D^2 \sqrt{h} \tag{49}$$

The value of K_1 depends upon the velocity V_1 and the area A_1. The former depends upon the value of c (Art. 83), and the latter depends upon the diameter D, the height of the runner B (Fig. 34), the value of the angle α_1, and also the number of buckets and guides.

Since there are so many factors involved, it will be seen that a given value of K_1 can be obtained in several ways. For some purposes it might be convenient to express these items by separate constants but for the present purpose it will be sufficient to cover all of them by the one constant.

The lowest value of K_1 will be obtained for the tangential water wheel with a single jet. For this type of wheel there is evidently no minimum value of K_1 below which we could not go. The maximum value of K_1 is, however, fixed by the maximum size of jet that may be used. (See Art. 30 and Art. 74.) Using this maximum size of jet we obtain a value of $K_1 = 0.0005$. However the more usual value is about $K_1 = 0.0003$. There is seldom any reason for using a large diameter of wheel with a small jet

050. These are not absolute limits but they cannot be exceeded very much and to do so at all would mean to extend our oportions of design beyond present practice. For the usual n of stock turbines values of K_1 vary from 0.005 to 0.025. To mmarize:

or the tangential water wheel $K_1 = 0.0002$ to 0.0005
or the reaction turbine $K_1 = 0.001$ to 0.050

107. Diameter and Power.—Since the power of any runner is oportional to the square of the diameter and to the three-halves ower of the head, we may write

$$h.p. = K_2 D^2 h^{3/2} \tag{50}$$

s the power is directly dependent upon the discharge it is ident that the discussion in the preceding article will apply ually well here. K_2 may be computed directly from K_1 if the ficiency is known, or it may be determined independently by st.

or the tangential water wheel $K_2 = 0.000018$ to 0.000045
or the reaction turbine $K_2 = 0.00008$ to 0.00450

108. Specific Speed.—In Art. 105 we have the relation between ameter and r.p.m.; in Art. 107 we have the relation between ameter and power. It is now desirable to establish the relaon between r.p.m. and power as follows:

rom (47)

$$D = \frac{1840\phi\sqrt{h}}{N}$$

rom (50)

$$\sqrt{K_2}D = \frac{\sqrt{h.p.}}{h^{3/4}}$$

ubstituting the above value of D in the second expression we ave

$$\sqrt{K_2}\frac{1840\,\phi\sqrt{h}}{} = \frac{\sqrt{h.p.}}{}$$

By then varying the diameter of the runner the value of will change in an inverse ratio, but the square root of the hor power varies directly as D. Thus the product of the two or remains constant for all values of D as long as the series is hon logous. If a value of D be chosen which will make the $h.p.$ 1.0 when $h = 1$ ft., we then have $N_s = N$.

That is, the specific speed is the speed at which a turb would run under one foot head if its diameter were such that would develop 1 h.p. under that head. The specific speed also an excellent index of the class to which a turbine belongs a hence the term type characteristic is very appropriate. Th is no standard symbol used by all to denote this constant thou N_s is quite common. Other notations are N_u, K_T, and numero others. In Europe the specific speed will be expressed in met units; to convert from one to the other multiply N_s in Engl units by 4.45.

It should be noted that the power to be used in this formula the power output of the machine. Thus the efficiency is involv in the value of N_s, though it does not appear directly. In t case of a Pelton wheel with two or more nozzles, the power to used is that corresponding to only one jet. In the case of mul runner units, the specific speed should be computed for t power of one runner.

For any turbine the value of N_s is a constant, so long as t speed of the turbine is varied as the square root of the hea For if N varies as $\sqrt{h}$ and the power varies as $h^{3/2}$, it is seen th $N\sqrt{h.p.}$ varies as $h^{5/4}$. Also for a series of homologous runners t square root of the power increases with D directly while the spe N varies inversely. Thus the factor is a constant for all turbi of the same type.

The value of the specific speed is ordinarily computed by equ

*$h^{5/4} = h \times h^{1/4} = h\sqrt{\sqrt{h}}$.

$$u_1 = \pi DN/720 = \phi\sqrt{2gh}$$

From which $\quad D = 720\phi\sqrt{2gh}/\pi N \qquad (52)$

Also, if $\quad B = mD,$

$$q = (0.95\pi BD/144)V_{r1} = 0.95\pi mD^2c_r\sqrt{2gh}/144 \qquad (53)$$

where 0.95 is a factor to compensate for the area taken up by the runner vanes.

Since $\quad B.h.p. = wqhe/550$

$$B.h.p. = 0.95\,w\pi\sqrt{2gm}\,D^2c_r.h^{3/2}\cdot e/144 \times 550 \qquad (54)$$

Eliminating D between the simultaneous equations (52) and (54) and reducing, we have (giving ϕ the special value ϕ_e)

$$N_s = \frac{N\sqrt{B.h.p.}}{h^{5/4}} = 252\phi_e\sqrt{c_r \times m \times e} \qquad (55)$$

This equation shows how the value of the specific speed may be varied in the design by means of the factors ϕ_e, c_r, and m.[1]

An instructive form, however, is that of Lewis F. Moody, in which the diameter of the draft tube is represented as nD, and the discharge velocity head $V_2^2/2g = Lh$, where L is the fractional part of the head h that is lost at discharge from the runner. (Of course an efficient draft tube is relied upon to recover a part of this). With these we may write

$$q = \frac{\pi(nD)^2}{4 \times 144}V_2 = \frac{\pi(nD)^2}{4 \times 144}\sqrt{L} \times \sqrt{2gh} \qquad (56)$$

Substituting this expression for q in that for horsepower, we obtain

$$B.h.p. = w\pi\sqrt{2g}n^2D^2\sqrt{L}he/4 \times 144 \times 550 \qquad (57)$$

Eliminating D between the simultaneous equations (52) and (57) and reducing, we have

$$N_s = \frac{N\sqrt{B.h.p.}}{h^{5/4}} = 129.5n\phi_e\sqrt{\sqrt{L}}\sqrt{e} \qquad (58)$$

[1]In a similar manner the specific speed for a Pelton wheel may be shown to be, $N_s = 129\phi_v\sqrt{c_v e}\frac{d}{D}$ where d = jet diameter in inches. Since, for the

increased, but it also will reach a definite limit, which is something under 1.0. The efficiency cannot readily be increased any more than for lower specific speed runners and as a matter of fact, is already decreasing. Thus after these factors have reached their maximum limits, so that they may be assumed to be constant, the only means of increasing N_s any further would appear to be by increasing L. Thus

$$N_s \propto \sqrt{\sqrt{L}} \text{ or } L \propto N_s^4 \tag{59}$$

But after this limit is passed so that equation (59) applies, the outflow loss increases much faster than the specific speed. Even with the best of draft tubes a certain percentage of L must be lost eventually and hence e is rapidly reduced. The outflow conditions thus impose a maximum limit upon N_s.

For the lower values of N_s the outflow loss becomes of small consequence, but other factors then enter. The chief of these are the leakage losses and the disk friction. For with small values of the specific speed the runner becomes relatively large in diameter and correspondingly narrow. The area of the spaces through which water can leak becomes of greater percentage as compared with the area through the runner. And the percentage of the power consumed in rotating the large diameter runner through the water in the clearance spaces becomes of increasing importance. If we assume that the power lost in disk friction varies as D^5N^3, it may be readily shown by combining this with equations (51) and (52) that the power so lost varies as ϕ^5/N_s^2. After ϕ_e has been reduced to its minimum, which approaches 0.50 as a limit, any further decrease in N_s increases the disk friction loss much more rapidly. Also as ϕ_e is decreased c_e must increase (approaching unity as a limit), as shown by equation (39), and consequently p_1 decreases (approaching zero as a limit). But this is undesirable, due to the danger of oxidation of parts of the runner.

In view of these facts, it may be shown that the minimum allowable value for the specific speed of a reaction turbine is about 10.

about 20 to 80.

A very recent type of turbine runner proposed by Nagler is of an axial flow type and is similar to a screw propeller. The present specific speed of this type is 165 and it is possible that this may be extended in the future.[1]

The impulse turbine runs in air and thus the disk friction loss for it becomes windage loss, which is of less consequence. There can be no leakage loss with this type and also the reduction of the pressure to atmospheric gives rise to no trouble. Hence this type of turbine is suitable for specific speeds below those for the reaction turbine. For the tangential water wheel there is no definite lower limit to its specific speed, save that the windage loss affects it in a similar manner to the disk friction in the case of the low-speed reaction turbine. But as the specific speed of a Pelton wheel is increased the size of the jet must become larger in proportion to that of the wheel and for the reasons already given there is a limit to this. The further increase in ratio of jet diameter to wheel diameter causes the efficiency to rapidly decrease, due to loss of water past the buckets. There have been cases of tangential wheels with specific speeds of less than 1 and maximum values of 6, though the latter involves some sacrifice of efficiency. The usual range in practice is from 3 to 4.5.

It will be seen that there is a gap in the values of N_s between the tangential water wheel and the reaction turbine. Similar gaps are also found for the values of ϕ_e, K_1, and K_2. In Europe a few two-stage radial inward-flow reaction turbines have been built and these could have lower values of the specific speed than 10. And by the use of two or more nozzles on one impulse wheel runner, the value of N_s for the tangential wheel can be increased above the 5 or 6 set as the limit for the single nozzle. Thus the entire field can be covered.

To recapitulate:

For the tangential water wheel	N_s = 3.5 to 4.5 (6 max.)
For the reaction turbine	N_s = 10 to 100.

heads. As these tables are supposed to ... they may be used for the determination of these factors. If all the runners of the series were strictly homologous it would be necessary to compute these constants for one case only. Actually variations will exist with different diameters of runners and thus there will be some variation in the values secured. Since each manufacturer usually makes several lines of runners so as to cover the field to better advantage, there will be as many distinct values of these constants as he makes types of runners. If the catalog tables are purely fictitious then the computations based upon them will not be very reliable.

110. Illustrative Case.—In order to illustrate the preceding article the following tables are given. For the sake of comparison only two firms out of many are chosen for this case. The values given are based upon catalog tables. Since K_2 depends upon K_1 it has been omitted to save space.

TABLE 3.—JAMES LEFFEL AND CO.

Type	ϕ	K_1	N_s
Standard................	0.722–0.727	0.0061–0.0064	30.8–32.6
Special....................	0.750–0.779	0.0094–0.0097	41.6–43.2
Samson....................	0.838–0.844	0.0170–0.0171	61.5–61.9
Improved Samson.........	0.856–0.886	0.0220–0.0220	71.0–73.5

TABLE 4.—DAYTON GLOBE IRON WORKS CO.

Type	ϕ	K_1	N_s
High head type...........	0.578–0.585	0.0051–0.0064	22.8–26.0
American.................	0.662–0.704	0.0054–0.0080	25.0–32.3
Special New American.....	0.697–0.727	0.0175–0.0205	50.0–57.4
Improved New American...	0.886–0.944	0.0233–0.0263	78.2–80.5

This table shows the variation in constants that might be expected, and shows also how each firm attempts to cover the ground. It will be noticed, however, that the two do not agree

design—it would not be a stock turbine, and would therefore be more expensive.

111. Uses of Constants.—After these factors are determined it will then be easy to find what results may be secured for any size turbine of the same design under any head. Another use for them is that when the limits are fixed they will enable one to tell what is possible and what is not. In the next chapter it will be shown how they are of direct use also in the selection of a turbine.

112. NUMERICAL ILLUSTRATIONS

1. The test of a 16-in. runner under a 25-ft. head gave the following as the best results: $N = 400$, $q = 17.5$ cu. ft. per second, h.p $= 39.8$. Find the constants.

$$\text{From (48) } \phi = 0.000543 \frac{16 \times 400}{5} = 0.696$$

$$\text{From (49) } K_1 = \frac{17.5}{16^2 \times 5} = 0.01368$$

$$\text{From (50) } K_2 = \frac{39.8}{16^2 \times 125} = 0.00124$$

$$\text{From (51) } N_s = \frac{400 \times 6.32}{55.8} = 45.2$$

2. Suppose that a 40-in. runner of the same design as in problem (1) is used under a 150-ft. head. Compute the speed, discharge, and horse-power.

$$\text{From (47) } N = \frac{1840 \times 0.696 \times 12.25}{40} = 392 \text{ r.p.m.}$$

From (49) $q = 0.01368 \times 1600 \times 12.25 = 268$ cu. ft. per second
From (34) $0.00124 \times 1600 \times 1838 = 3650$ h.p.

3. Suppose that turbines of the type in problem (1) were satisfactory for a certain plant but that the number of the units (and consequently the power of each) and the speed has not been decided upon. If the head is 150 ft., then by (51)

$$N \times \sqrt{h.p.} = 45.2 \times 525 = 23{,}730.$$

By the use of different diameters of runners of this one type the following results can be secured:

14,100 h.p. at 200 r.p.m.
6,250 h.p. at 300 r.p.m.

300 r.p.m., 16 units at 600 r.p.m., or 36 units at 900 r.p.m. If none of the possible combinations were suitable it would be necessary to use another type of turbine—that is one with a different value of N_s.

By equation (50) the diameters are found to be 52.3 in., 26.2 in., and 17.5 in. for 300, 600, and 900 r.p.m. respectively.

4. Compute values of ϕ, K_1, K_2, and N_s for each of the turbines whose tests are given in Appendix C: (a) for the point of highest efficiency, (b) for the point of maximum power.

113. QUESTIONS AND PROBLEMS

1. How do the speed, rate of discharge, power, and efficiency of a turbine vary with the head, the value of ϕ remaining constant? Why?

2. Suppose the speed of a turbine remains constant while the head changes, how will the rate of discharge, power and efficiency vary? What is necessary in order to answer this question?

3. How do the speed, power, and efficiency vary with the diameter of a series of homologous runners? Why? How do these quantities change when both the head and diameter are different, the runners being of the same type, however?

4. What is the physical meaning of the term "specific speed?" Why are the terms "type characteristic" and "characteristic speed" also appropriate? How may the value of this factor be changed in the design of the runner?

5. What limits the maximum and minimum values of the specific speed for reaction turbines? For impulse wheels? Why do the latter have lower specific speeds than the former?

6. If a turbine gives an efficiency of 82 per cent. when tested under a head of 10 ft. what would you estimate its efficiency to be if installed under a head of 100 ft.? Under a head of 225 ft.? If the test of a 27-in. runner under a head of 150 ft. gives, as the best results, $N = 600$, $q = 40$, h.p. = 550, what will be the speed, rate of discharge, and power of a 54-in. runner of the same type under a head of 50 ft.?

7. If a turbine is desired to run at 300 r.p.m. under a head of 60 ft., what are the minimum and maximum diameters of runners that might be used? If 30 cu. ft. of water per second is to be used under a head of 60 ft., what range of diameters might be employed?

8. Suppose that a type of turbine, whose specific speed is 80, is suitable for use in a certain plant where the head is 16 ft. What combinations of h.p. and r.p.m. are possible?

9. If a tangential water wheel was desired to deliver 1000 h.p. under

nes shown in Fig. 54, making whatever assumptions are

whose dimensions are given in problem (11) had a value the per cent. of the total head that is equal to the velocity rom the runner?

CHAPTER XII

TURBINE CHARACTERISTICS

114. Efficiency as a Function of Speed and Gate Opening. In Fig. 87, page 112, it has been shown how the power, and hence the efficiency, of an impulse turbine varies with the speed for

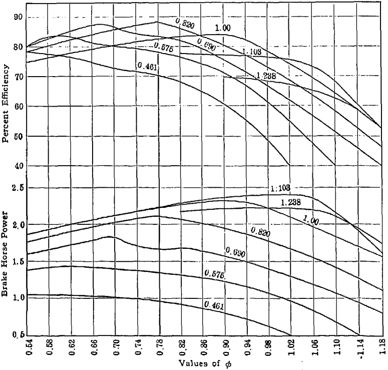


FIG. 107.—Characteristics of high-speed runner. $N_s = 93$.

any gate opening; and in Fig. 91, page 115, how the efficiency varies with the power at different gate openings at a uniform

thing less than "full" gate.[1] Therefore, in general, the maximum efficiency and the maximum power are found at different gate openings and different speeds.

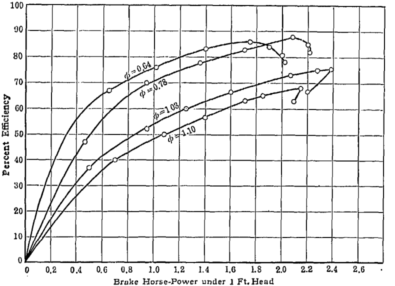


FIG. 108.—Efficiency-power curves for different speeds under same head.

Thus in Fig. 107 the maximum value of the efficiency is found at 0.820 gate and at such a speed that $\phi_e = 0.780$, but the maximum power is found at 1.103* gate and at such a speed that $\phi = 1.03$. The efficiency in the former case is 0.88 and in the latter 0.77. In Fig. 108 are shown efficiency curves as a function of power for values of $\phi = 0.64$, 0.78, 1.03 and 1.10.

[1] This statement does not hold in the case of the cylinder gate turbine, where maximum power and maximum efficiency coincide at full gate, but this type is of little importance at present.

* The numbers indicating the extent of the gate opening are purely arbitrary and 1.0 does not necessarily indicate the maximum gate opening.

if the load is variable and especially if it is apt to be light for long periods of time a lower value of the speed might give a higher average efficiency, though the peak is not so high. On the other hand it may be deemed worth while to sacrifice efficiency for the sake of capacity and increased speed, which could be attained by using the higher values of the speed. It should be borne in mind that some of these results might be better attained with another type of turbine, but the latter is a subject for consideration in

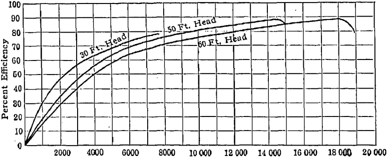


FIG. 109.—Power and efficiency of a turbine at constant speed under different heads.

the next chapter. We are here studying the possibilities of a single turbine or at least a single type of turbine.

It must be remembered, as explained in Art. 20, that the head is apt to vary for many water power plants, especially those under low head. If the head decreases in time of flood, the power output of the turbine may be seriously reduced. Under these circumstances the important consideration is the maximum power output. Since there is a superabundance of water for the time being, efficiency is a secondary consideration. While efficiency under the normal head is of importance, it might be sacrificed to some extent in favor of a speed which would be such as to give the maximum power under flood conditions. It is

the speed for the normal head, the results under all other conditions will be altered, and careful study must be made of all the variables to decide what is best.

115. Specific Speed an Index of Type.—Both the elements of speed and capacity are involved in the specific speed. It was stated in Art. 38 that both speed and capacity were merely relative terms; that is, a high-speed turbine is not necessarily one which runs at a high r.p.m., but one whose speed is high compared with other turbines of the same power under the same head. In like manner a high-capacity turbine is not necessarily one of great power but merely one whose power is high compared with others at the same speed under the same head. Since $N_s = \frac{N\sqrt{h.p.}}{h^{5/4}}$ it is evident that a low-speed, low-capacity turbine will be indicated by a low value of N_s and a high-speed high-capacity turbine by a high value of N_s. As stated in Art. 108, values of N_s for the tangential water wheel may run up as high as 5 or 6, for the reaction turbine they range from 10 to 100. Values in the neighborhood of 20 indicate a runner such as Type I in Fig. 34, while values in the neighborhood of 80 indicate Type IV. Thus when the speed and horsepower of any turbine under a given head are specified the type of turbine necessary is fixed.

Other things being equal, it is seen that a high head means a comparatively low value of N_s while a low head means a high value. Aside from any structural features it is apparent that a high head calls for a tangential water wheel or a low-speed reaction turbine, while a low head demands a high-speed reaction turbine. However, the head alone does not determine the value of N_s. So far as the r.p.m. is concerned there may be considerable variation, yet neither a very low nor a very high r.p.m. is desirable and for the present purpose we may suppose that it is restricted within narrow limits. The value of N_s will thus be affected by the power of the turbine as well as the head. If

and h.p. that may be substituted in equation (87), with a resulting variety of values of N_s for the particular turbine. It is thus necessary to define the speed and power for which this factor is to be computed, if it is to have a definite value for a given runner. The current practice is to rate turbines at the maximum guaranteed capacity, the actual maximum capacity being usually slightly greater than this, since the builder allows a small margin to insure his meeting the guarantee. The *nominal* specific speed is that corresponding to this rated capacity at a stated speed. But under a given head the turbine speed might be selected from a limited range of values, as explained in Art. 114. It may be seen that, though the true maximum power of the turbine is a definite value, the actual maximum power it can deliver at full gate, under the operating conditions, depends upon the speed at which it is run. Hence the value of N_s, as thus computed, varies with the speed, and is not a perfectly definite value. Despite this, the value of specific speed is usually so computed because the rated capacity is often known when the power and speed for maximum efficiency are not.

For accurate comparisons of one turbine with another and for exact work, it is best to select the values of power and speed for which the true maximum efficiency is obtained. The value of N_s, so computed, may be called the *true* specific speed. Since this is based upon a single definite point, there can be but one value for the turbine.

116. Illustrations of Specific Speed.—For a turbine of 2000 h.p. at 1000 r.p.m. under 1600 ft. head the value of N_s is 4.42. Thus a very low-speed turbine, the tangential water wheel, is required. The actual r.p.m., however, is high.

For a 5000 h.p. turbine at 100 r.p.m. under 36 ft. head N_s equals 80.3. Thus a high-speed reaction turbine is indicated, though the actual r.p.m. may be relatively low.

Suppose that a 12-h.p. turbine is to be run at 100 r.p.m. under a 36-ft. head, the value of the specific speed is 3.95, which means a tangential water wheel. For the larger power under the same

If 10,000 h.p. is required at 300 r.p.m. under a 60-ft. head, the value of N_s would be 179.5. As this is an impossible value it would be necessary to reduce the speed or to divide the power up among at least 4 units of 2500 h.p. each.

117. Selection of a Stock Turbine.—The choice of the type of turbine will be taken up in the next chapter. For the present suppose that required values of speed and power under the given head are determined. The value of the specific speed can then be computed and will indicate the type necessary. If the turbine is to be built as a special turbine nothing more is to be done except to turn the specifications over to the builders.

If, however, the turbine is to be selected from the stock runners listed in the catalogs of the various makers, it will be necessary to find out what firms are prepared to furnish that particular type of runner. It would be a tedious matter to search through a number of tables in numerous catalogs to find the particular combination desired, but the labor is avoided by the use of the constants given in the preceding chapter. It will be necessary merely to compute values of specific speeds of turbines made by different manufacturers. This can be quite readily done and such a table will always be available for future use.

A make of turbine should then be selected having a value of N_s very near to the value desired. The value of N_s ought to be as large as that required, otherwise the turbine may prove deficient in power, and for the best efficiency under the usual loads it should not greatly exceed the desired value. Having selected several suitable runners in this way, bids may be called for. These bids should be accompanied by official signed reports of Holyoke tests of this size of wheel or the nearest sizes above and below, if none of that particular size are available. This is to enable us to check up the constants obtained from catalog data and to verify the efficiencies claimed. Holyoke test data is very essential if the conditions of the installation are such that an accurate test is not feasible. In making a final choice other

Maker	Type	N_s	K_e
Camden Water Wheel Wks...	United States Turbine.....	64.7	0.00190
James Leffel and Co.........	Samson..................	61.8	0.00158
Platt Iron Works...........	Victor Standard..........	63.0	0.00205
Trump Mfg. Co............	Standard Trump..........	61.5	0.00210

It is thus apparent that any one of these manufacturers could supply a turbine from their present designs which would nearly fill the requirements. A number of other firms in this country could not fit the case except with a special design or a modification of an existing design. Thus inspection of the table for the Dayton Globe Iron Works Co. in Art. 110 will show that the nearest approach they have to it is their Special New American with an average value of N_s of 53.7. They could supply a turbine to run at 120 r.p.m. under the head specified, but it would develop only 358 h.p. Or if they supplied a turbine capable of delivering 480 h.p. it should run at 103.5 r.p.m.

Turning to the four cases presented in the table, it is apparent that the Camden wheel is a little over the required capacity, but it may not be enough to be objectionable. The Platt Iron Works wheel is very little over the required capacity and the Leffel and Trump are a trifle under it. If there is a little margin allowable in the power, any of these might be used. The value of N_s according to which the wheel is rated should be the value for the speed and power at which it develops its best efficiency. In any plant the variation in the head produces a deviation from the best value of ϕ, if the wheel be run at constant speed, and thus causes a drop in efficiency. The power of the wheel may increase or decrease according to the way the head changes. Thus in actual operation the conditions depart so much from those specified in the determination of N_s that small discrepancies in its value such as exist in the table are of little importance.

e attempt to do is merely to select a turbine the peak of whose fficiency falls as near as possible to the conditions of head, peed, and power chosen. Al-ough our conditions may be ch that we may rarely realize e very highest efficiency of hich the turbine is capable, et we should be very close it.

119. Variable Load and ead.—In any plant the load usually not constant but aries over a considerable nge. In comparing turbines r certain situations the aver-ge operating efficiency may e more important than the fficiency on full load only. the turbine is to run on full ad most of the time or if the stallation is such that the ndage is limited or lacking together, then the efficiency part load is of little im-rtance. But if the load is riable and if water can be ored up during the time the heel may be running under a ht load then the efficiency at times becomes of interest. the plant has a number of its it is possible to shut down some of them at times so as to ep the rest on full load.

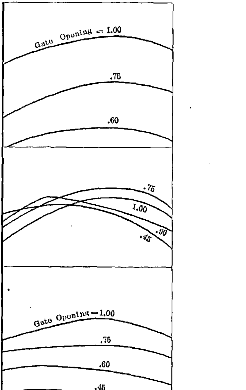


FIG. 110.

In most low head plants the variation in head is a serious item

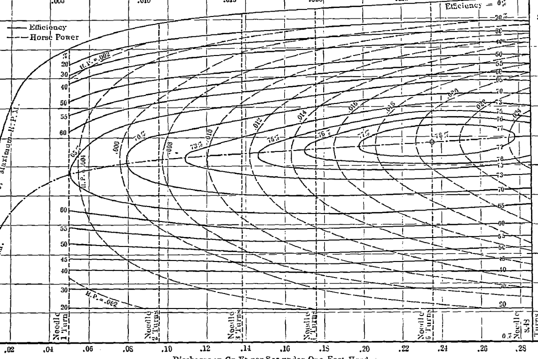


FIG. 111.—Characteristic curve for a 24″ tangential water wheel.

will be the chief item.

These factors can be studied by means of curves such as are shown in Fig. 110. Efficiency, discharge, and power for various gate openings reduced to 1-ft. head are plotted against ϕ or the r.p.m. under 1-ft. head. The normal speed and power should be that corresponding to the maximum efficiency. If the wheel is run at constant speed a variation in head causes a change in ϕ.

120. Characteristic Curve.—For a thorough study of a turbine the characteristic curve is a most valuable graphic aid. The coordinates of such a curve are discharge under 1-ft. head and ϕ or r.p.m. under 1-ft. head. Values of the horsepower input under 1-ft. head should also be laid off to correspond to the values of the discharge. Lines should then be drawn on the diagram to indicate the relation between speed and discharge for various gate openings. Alongside of each experimental point giving this relation, the value of the efficiency should be written. When a number of such points are located, lines of equal efficiency may be drawn by interpolation.

Another very good method is to draw curves of efficiency as a function of ϕ for each gate opening. For any iso-efficiency curve desired on this diagram it is possible to read off corresponding values of ϕ.

If desired, lines of equal power may also be constructed. To do so, assume the horsepower of the desired curve, then compute the horsepower input for any efficiency by the relation, horsepower input = horsepower output $\div$ e. This value of e on one of the iso-efficiency curves together with the value of horsepower input locates one or two points of an iso-power curve.

The characteristic curve for a 24-in. tangential water wheel is shown in Fig. 111. This curve covers all the possible conditions under which the wheel might run. The only way to extend the field would be to put on a larger nozzle. Since the discharge of a tangential water wheel is independent of the speed the lines for the various gate openings will be straight. For the reaction

Engineering." Other diagrams have also been proposed by various men, the object of all being to represent the fundamental variables in the best form for the ready comparison of one turbine with another.

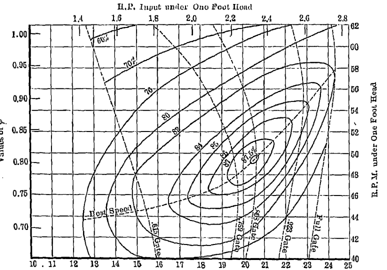


FIG. 112.—Characteristic curve for a high speed reaction turbine.

121. Use of Characteristic Curve.—From the characteristic curve it is apparent, at a glance, at what speed the turbine should run for the best efficiency at any gate opening. The best efficiency in Fig. 111 is obtained when $\phi = 0.457$ or $N_1 = 34$, and with the needle open 6 turns. With full nozzle opening the best value of N_1 is 35, with the needle open 3 turns the best speed is such that $N_1 = 32$. (With the reaction turbine these differences would be greater.)

From the characteristic curves any other curves may be constructed. For constant speed follow along a horizontal line, for a fixed gate opening follow along the curve for that relation.

is maintained at 275 r.p.m. when the head is 74 ft., then $\phi = 0.429$ and $N_1 = 32$. If $h = 55$ ft., $\phi = 0.497$ and $N_1 = 37$. In the last case the best efficiency would be 77 per cent., a drop of 1 per cent.

The iso-efficiency curves represent contour lines on a relief model and thus the point of maximum efficiency is represented by a peak. It is apparent that for varying loads or heads a turbine giving a diagram, that indicates a model with gentle slopes from this point, would probably be better than a turbine for which the peak might be higher and the slopes steeper. The relative increase in discharge capacity at full gate as ϕ increases is also apparent and indicates which turbine is better for operation at reduced head but normal speed.

122. QUESTIONS AND PROBLEMS

1. For any turbine, how does the speed for highest efficiency vary with the gate opening used? How does the efficiency vary with the gate opening for any speed? At what speed and gate will maximum efficiency be found, as compared with maximum power?

2. Should a turbine necessarily be run at the speed for maximum efficiency? Why?

3. What happens to the power and efficiency of a turbine when the head changes, but the speed is kept constant? In time of flood, what is the important consideration?

4. What is the difference between the true and the nominal specific speed? What would be the general profile of a runner whose specific speed was 10? What of one whose specific speed was 100?

5. When is efficiency on full load important and when is efficiency on part load of more value? When is maximum power of principal interest? When is maximum speed the chief object?

6. If a plant contains a number of units, what should be done if all of them are carrying half load? Why? Would there be any object in shutting down some of them if the supply of water was abundant?

7. If there is a great shortage of water so that the supply is inadequate for all the wheels at full head, so that the water level falls considerably below normal before equilibrium is attained, is it better to operate the plant with all the wheels or shut down enough of them to keep the water level near the

output and what the power at the point of maximum efficiency? If the speed is kept at this same value while the head falls to 36.5 ft., what will be the value of the maximum power delivered?

Ans. 755 h.p., 720 h.p., 498 h.p.

10. In problem (9) the second value of the head is 73 per cent. of its initial value and the maximum power is 66 per cent. of its value in the first case. For the impulse turbine in Fig. 111, what would be the ratio of the maximum power outputs, if the head dropped the same proportional amount while the speed remained the same as for the maximum efficiency under the initial head? *Ans.* 58.7 per cent.

11. Suppose that an impulse wheel, similar to that for which the curves of Fig. 111 were drawn, is made of such a size as to develop 5000 h.p. under a head of 1200 ft. Find the diameter of the wheel, its r.p.m., and plot a curve between efficiency and power for a constant speed.

CHAPTER XIII

SELECTION OF TYPE OF TURBINE

123. Possible Choice.—It has been shown that, if the speed d power under a given head are fixed, the type of turbine cessary is determined. If there is some leeway in these mats it may be possible to vary the specific speed through a nsiderable range of values. Suppose turbines of a given power y be run at 120 r.p.m., at 600 r.p.m., or at 900 r.p.m. Each e of these would give us a different specific speed and thus a ferent type of runner. Or, if the speed be fixed, the power, ch as 20,000 h.p. may be developed in a single unit, in two its of 10,000 h.p. each, or in eight units of 2500 h.p. each. ain we have different types of runners demanded. Both the eed and power may be varied in some cases and the choice is der still. As an example, it may be required to develop) h.p. under 140-ft. head. Suppose this power is to be divided between two runners and the speed to be 120 r.p.m. The ue of N_s is then 4.12, showing that a double overhung tanntial water wheel is required. Or if the power be developed a single runner at 600 r.p.m., the value of N_s would be 29.2, ich would call for a reaction turbine.

It is customary to choose a speed between certain limits, as ther a very low nor a very high r.p.m. is desirable. Also the mber of units into which a given power is divided is limited. vertheless considerable latitude is left. It remains to be seen at considerations would lead us to choose such values of speed d power as would permit the use of a certain type of runner.

124. Maximum Efficiency.—The best efficiency developed a turbine will depend, to some extent, upon the class to which belongs. The impulse and reaction turbines are so different their construction and operation that the difference in efficiency between them can be determined solely by experiment. owever, abstract reasoning alone will lead to certain conclusions to the relative merits of different types within each of these

as the normal type. On the other hand too low a specific speed is not conducive to efficiency, since the diameter of the wheel becomes relatively large in proportion to the power developed, so that the bearing friction and windage losses tend to become too large in percentage value. The value of N_s for the highest efficiency is about 4.

A low specific speed reaction turbine, such as Type I in Fig. 34 for example, will have a small value of the angle α_1. A consideration of the theory, especially equation (33), shows that this is conducive to high efficiency. However this is more than offset by other factors, such as the large percentage value of the disk friction, as explained in Art. 108. In addition, the leakage area through the clearance spaces becomes a greater proportion of the area through the turbine passages, and also the hydraulic friction through the small bucket passages is larger. The result of all these factors is that the efficiency tends to be reduced as very small values of the specific speed are approached.

A medium specific speed turbine runner would have a somewhat larger value of the guide vane angle but this slight disadvantage would be more than offset by the reduction in the relative values of the disk friction, leakage loss, and hydraulic friction loss within the runner. Thus this type would have a higher efficiency than the former.

But when the high specific speed type is reached the inherently large value of the discharge loss is such as to materially reduce the efficiency. This reduction is aided also by the large value given to the guide vane angle and opposed by the decreased disk friction, leakage through the clearance spaces, and internal hydraulic friction. However the effect of these latter factors is not sufficient to offset the increased discharge loss. In other words, efficiency has been sacrificed in favor of increased speed and capacity, just as in the case of a high-speed impulse turbine.

This reasoning is borne out by the facts, as can be seen by Fig. 113, where efficiency is plotted as a function of specific speed.[1] A number of test points were located and th

that one should expect in every case. It merely shows the relative merits of the different types. The actual efficiency obtained depends not only upon the specific speed but also upon the capacity of the turbine and the head and other factors. The larger the capacity of a turbine the higher the efficiency will be. In a given case the efficiency obtained for a specific speed of 30, say, might be only 83 instead of the 93 shown by the curve. But if the specific speed had been 95 instead of 30 the efficiency realized might have been only 73.

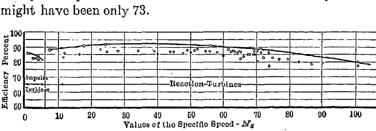


FIG. 113.

Higher efficiencies have been attained with reaction turbines than with Pelton wheels. The maximum recorded efficiency for the former is 93.7 per cent. and quite a few large units have shown efficiencies over 90 per cent. where conditions were favorable. The highest reported value for an impulse wheel is 89 per cent. but the usual maximum is about 82 per cent. However the efficiency of a reaction turbine is a function of its capacity, that is for small sizes the efficiency is relatively low. As the larger sizes are reached this difference disappears. The reason for this is that the clearance spaces and hence leakage losses are a greater percentage with the small sizes. The efficiency of the Pelton wheel is not dependent on its size. Hence for smaller powers the tangential wheel may have a higher maximum efficiency than the reaction turbine.

125. Efficiency on Part-load.—Full-load will be defined as the

as possible. In order to obtain the former the vane an
must agree with the angle of the relative velocity of the w
determined by the vector diagram, and the quantity of
should be such that its relative velocity v'_1, as determined
equation of continuity, should agree with the velocity
determined by the vector diagram of velocities. In order
duce the discharge loss to a minimum it has also been show
α_2 should have a value of approximately 90°.

There is practically no additional loss at entrance to the
ets of a Pelton wheel due to the reduction in the size of
at part-load. If the jet and wheel velocities remained ju
same, the velocity diagrams would be identical at all
Actually the jet velocity may vary slightly but the shape
buckets is such that there is no well defined vane angle
trance. And since, in the impulse turbine, the relative v
through the runner is not determined by the equation
tinuity, there can be no abrupt change in either the direc
magnitude of the relative velocity of the water at entrance.
this is not the case with the reaction turbine. The small
opening changes the angle α'_1. This alters the entrance v
diagram. Hence the angle β'_1 will no longer agree with th
angle β'_1. Since the quantity of water discharged per un
is less than before, it follows that the velocity v'_1, as deter
by the area of the runner passages, is less than the value
load. Thus when a reaction turbine runs at part-gate th
eddy losses produced at entrance to the runner due to the
change in the direction and magnitude of the velocity
water through the wheel passages. No such losses occur w
impulse turbine.

At the point of discharge the velocity diagram for the tan
wheel is practically the same at all loads, provided the jet v
and bucket velocity are the same. There may be slight in
in the losses in flow over the bucket surfaces which would
this statement somewhat for very large or very small

opening, and hence the greater the effect produced upon the efficiency when this loss is increased at part-gate.

It is thus apparent that at part-load there are inherent losses within the reaction turbine that are not found with the Pelton wheel.

In fact the hydraulic efficiency of the latter would appear to be the same at all nozzle openings. In reality the reduction in the velocity coefficient of the nozzle, as the needle closes the discharge area, together with some change in the bucket friction, changes the efficiency slightly. It is the gross efficiency with which we are really concerned, and of course the mechanical losses due to friction and windage, which are constant at constant speed, cause the efficiency to decrease as the gate opening decreases. But the efficiency-load curve of the tangential water wheel is inherently a flat curve.

The losses within the reaction turbine runner are such that the hydraulic efficiency must decrease as the gate is changed in either direction from the position at full-load. Hence the efficiency at part-load or overload tends to be less than that for the impulse wheel, as shown in Fig. 114 (assuming both to be the same at full-load), and the higher the specific speed the steeper will the efficiency curve be.

For the tangential water wheel in Fig. 111 it can be seen that the best speed is slightly different for different gate openings and that it increases as the latter increases. This is also true with the reaction turbine, but in a more marked degree as can be seen in Fig. 112. If the speed is selected so as to give the best efficiency at a certain gate opening it will not be correct for any other gate opening and thus efficiency will be sacrificed at all gates except one.

This variation of the best speed with different gate openings is found in all turbines, but not in the same degree. With the low-speed reaction turbine it is small, approaching the tangential water wheel in that regard. With the high-speed reaction tur-

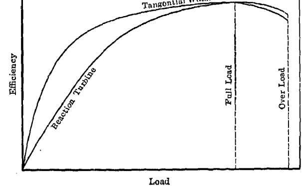


FIG. 114.—Relative efficiencies on part-load of impulse and reaction turbines.

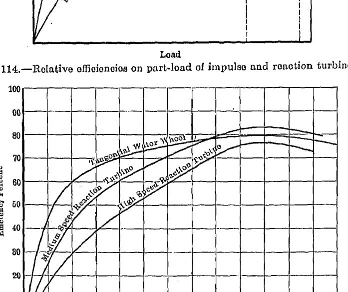


FIG. 115.—Typical efficiency curves.

sacrifice will be greater the higher the specific speed of the turbine. These considerations, together with the facts given in the preceding article, imply efficiency curves for the various types

It may also be noticed that there is less overload capacity with the high-speed turbine than with the other types. This is because the point of maximum efficiency is nearer full-gate than with the other types. If the customary 25 per cent. overload must be allowed, then the normal load must be less than the power for maximum efficiency with a further decrease in operating efficiency.

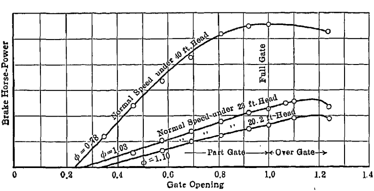


FIG. 116.—Relation between power and gate opening for same speed under different heads.

Thus from the tangential wheel on the one hand to the high-speed reaction turbine on the other the relative efficiency on part-load decreases as the specific speed increases.

126. Overgate with High-speed Turbines.—With the wicket or swing gates, as used today, there is no definite limit to their opening save that imposed by an arbitrary mechanical stop. As the gate opening increases the rate of discharge and hence the power of the turbine increases, as shown by curve for $\phi = 0.78$ in Fig. 116. But with too great an angle of the vanes the efficiency decreases so much that the power output no longer continues to increase and may even decrease. Ordinarily there is no advantage gained by opening the gates any wider than that necessary to secure maximum power, and hence the mechanism is usually

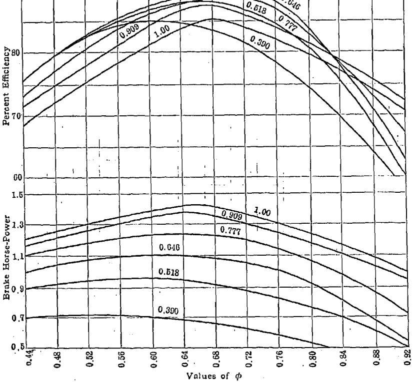


FIG. 117.—Characteristics of low-speed runner. $N_s = 27$.

If the normal speed be taken as the speed at which the wheel develops its maximum efficiency, it may be seen in Fig. 112 that

[1] This is a purely arbitrary definition, but there is at present no agreement as to what the term "full-gate" really signifies, and so it will be here used as defined above. It may be noted that the gate movement might also be limited to something less than the position shown, and in such an event it would be logical to denote the maximum opening as the "full-gate." The effect of this construction would be to decrease the overload capacity or to move the power for maximum efficiency nearer to the maximum power

with the low specific speed turbine at full gate, if the speed is increased above normal. The explanation of this difference in the two types is that with the inward flow turbine the centrifugal action opposes the flow of water, and hence the rate of discharge tends to decrease as the speed increases, while with the outward flow turbine the centrifugal action tends to increase the rate of discharge with the speed. This may be seen in Fig. 95, page 126. The low specific speed runner approaches the pure radial inward flow type, while the high specific speed runner of the present with inward, downward, and outward flow (the radius to the outer limit of the discharge edge being often as much as one-third greater than the radius to the entrance edge) approaches the outward flow turbine in this characteristic. Thus, despite the decrease in efficiency, as the speed departs from the normal, the increased rate of discharge tends to increase the power output for a certain range of speed above normal. This feature of the high-speed turbine is of great value, as it especially fits it for the class of service, to which it is otherwise adapted.

As has been explained, the maximum opening of the turbine gate would usually be that at which no further increase in power at normal speed could be obtained and this is termed "full-gate." But with the high specific speed turbine it is found that, when running at a speed above normal, the power continues to increase for an opening of the gate beyond its usual maximum value, as shown by curves for $\phi = 1.03$ and 1.10 in Fig. 116. A turbine so constructed that the gate can be opened wider than the maximum value necessary under normal conditions is said to be "overgated." This additional gate opening would be of no value with a low-speed turbine under any circumstances, and it would be of no value with a high-speed turbine under normal conditions. But, not only does the power of the latter increase at full-gate for speeds higher than the normal, but by opening the gate wider than the usual value the power may be still further increased as may be seen in Figs. 107 and 118. The nominal full-gate opening is denoted by unity.

proportional to $\phi\sqrt{h}$, a constant speed under a reduced head means an increase in ϕ above its normal value. As has been seen,

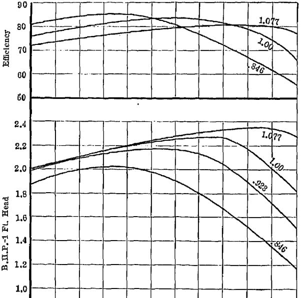


FIG. 118.—High-speed turbine.

an increase in ϕ above its normal value causes no increase in the power of a low- or medium-speed runner, but with the high-speed runner not only does the power at full-gate increase but, by overgating, the power may be still further increased. The effect of an overgate is to materially increase the capacity of the turbine at a time when there is a shortage of power. This feature is not possessed by lower speed runners.

plant and, as has been seen, the characteristics of the high-speed runner are such that it is able to deliver more power under these circumstances.

The results obtained with "overgating" high-speed turbines may be seen in Figs. 107 and 118. In the latter case full-gate is denoted by 1.00 and the maximum gate opening by 1.077. The normal speed is 52 r.p.m. under 1 ft. head. At that speed any further gate opening would be of no advantage, and in fact would merely cause a drop in efficiency. But at a higher speed, such as 65 r.p.m., the overgate feature raises the power under 1ft. head from 2.0 h.p. at full-gate to 2.35. Bearing in mind that a medium speed runner for the same situation would deliver less than 2 h.p. under these circumstances, it is seen how much superior the high specific speed turbine is for the particular conditions of service.

The differences between the low- and high-speed runners are brought out in the following table. The normal head is 15 ft. and the wheels develop 100 h.p. In time of high water the head will decrease to 10 ft., while the wheels are kept at their normal speed.

TABLE 6.—HEAD = 15 FT.

Type	N_s	R.p.m.	ϕ	Gate	H.p.	Efficiency
Low speed............	35.2	104	0.70	1.00	100	85
High speed...........	77.7	232	0.81	1.00	100	82

TABLE 7.—HEAD = 10 FT.

Type	N_s[1]	R.p.m.	ϕ	Gate	H.p.	Efficiency
Low speed............	35.2	104	0.86	1.00	47.3	78
High speed...........	77.7	232	0.99	1.00	49.3	76
High speed...........	....	232	0.99	1.077	58.0	80

[1] These values of specific speed apply only when the turbine is developing

true, the value of the head does exert a predominating influen
and hence there is some justification for the presentation of
relationship between the two, such as is given in Table 8. Ho
ever, it should be noted that the figures given for specific spe
are merely limits. Thus a head of 100 ft. does not require
turbine whose specific speed is 50 for example. The latt
is merely the maximum value found in current practice for such
head, and a lower value of N_s might be used. Within this ma
imum limit the specific speed chosen would depend upon t
power and speed and a consideration of the characteristics desire

TABLE 8.—RELATION BETWEEN HEAD AND SPECIFIC SPEED

Head ft.	Maximum value of N_s	Type of setting
20	100	Vertical shaft single runner units w good draft tubes.
25	90	
35	80	
50	70	
65	60	Single runner, either horizontal or v tical, or two runners discharging into common draft chest.
100	50	
160	40	
350	30	Single or double discharge runner horizontal shaft.
600	20	
800	10	
1000	6	Impulse wheels.
2000	3	
5000	1	

128. Choice of Type for Low Head.—No definite rules can
laid down for universal use because each case is a separate pro
lem. Neither is it possible to draw any line between a high a
a low head. All that can be done is to assume cases that a
typical and establish broad general conclusions. In any partic

being discharged over the spillway of the dam. Economy of water on part-load is thus of very little importance. The efficiency on full load is of value as it determines the amount of power that may be developed from the flow available.

Under a low head the r.p.m. is normally low and it is desirable to have a runner with a small diameter and a high value of ϕ, in order to secure a reasonable speed. A high speed means a cheaper generator and, to some extent, a cheaper turbine. These were the factors that brought about the development of the high-speed turbine.

A low-head plant is also usually subjected to a relatively large variation in the head under which it operates. When the head falls below its normal value the overgate feature of the high-speed turbine, enabling it to hold up the power, to some extent, at a good efficiency, is a very valuable characteristic.

The only disadvantage of the high-speed turbine for the typical low-head plant is that its maximum efficiency under normal head is not as good as that of the lower speed turbines. However, the other advantages outweigh this so that it is undoubtedly the best for the purpose.

129. Choice of Type for Medium Head.—With a somewhat higher head a limited amount of storage capacity usually becomes available and thus the efficiency on part-load becomes of interest as well as the efficiency on full-load. The r.p.m. also approaches a more desirable value so that the necessity for a high-speed runner disappears. The variation in head will generally be less serious also, so that the overgate feature of the high-speed turbine becomes of less value. The high efficiency of the medium-speed turbine fits it for this case. The high-speed turbine should not be used unless the interest on the money saved is more than the value of the power lost through the lower efficiency.

130. Choice of Type for High Head.—For high heads the possibility of extensive storage increases and the average operating efficiency then becomes of more interest than the maximum efficiency, especially if the turbine is to run under a variable load.

place. If the load is apt to vary over a wide range and be very light a considerable portion of the time, the comparatively flat efficiency curve of the tangential water wheel renders it suitable There is little difference between the characteristics of the low and medium-speed wheels. The choice between them is largely a matter of the r.p.m. desired, although there is some slight difference in efficiency.

131. Choice of Type for Very High Head.—Within certain limits there is a choice between the low-speed reaction turbine and the tangential water wheel. The former might be chosen in some cases because of its higher speed with a consequently cheaper generator and the smaller floor space occupied by the unit. The latter has the advantage of greater freedom from breakdowns and the greater ease with which repairs may be made. This consideration is of more value with the average high-head plant than with the average low-head plant, since the former is usually found in a mountainous region where it is comparatively inaccessible, and is away from shops where machine work can be readily done.

For extremely high heads there is no choice. The structural features necessary are such that the tangential water wheel is the only type possible. Also the relatively low speed of the tangential water wheel is of advantage where the speed is inherently high.

132. QUESTIONS AND PROBLEMS

1. For a given head and stream flow available at a certain power plant, what quantities may be changed so as to permit the use of various types of turbines? Which type of turbine will give the smallest number of units in the plant? Which type will run at the lowest r.p.m.?

2. How do impulse wheels and reaction turbines compare as to the maximum efficiency attained by each? How does the efficiency of an impulse wheel vary with its size? Why? How does that of a reaction turbine vary with its size? Why?

3. For the same power under the same head compare impulse wheels and reaction turbines with respect to efficiency, rotative speed, space occupied, freedom from breakdown

Why?

7. What is meant by full-load? What affects the efficiency of a tangential water wheel on part-load?

8. What affects the efficiency of a reaction turbine on part-load? Is the part-load efficiency a function of specific speed?

9. What is meant by full-gate? By overgate? What types of turbines are overgated?

10. What is the difference in the characteristics of low and high specific speed reaction turbines when run at the same speed under a head less than normal? Why?

11. What are the advantages and disadvantages of very high specific speed turbine runners?

12. What types of turbines could be used under a head of 20 ft.? Under 200 ft.? Under 1000 ft.?

13. What are the advantages of a high-speed runner under very low heads? What are the advantages of a medium speed runner under the same conditions?

14. What are the especial merits of tangential water wheels for very high heads? What are the disadvantages of a low-speed reaction turbine for the same conditions?

15. The turbine runner for which the curves in Fig. 107 were plotted was 23 in. in diameter and had a specific speed of 93. The specific speed of the runner for which the curves of Fig. 117 were drawn was 27 and the diameter was 57 in. Suppose a turbine was required to deliver 1200 h.p. at full-gate under a head of 25 ft., find the size and r.p.m. for a runner of each of these types. *Ans.* 47.8 in., 150 in., 144 r.p.m., 43.5 r.p.m.

16. If the speeds remain as in problem (15) while the head decreases from 25 ft. to 16 ft., find the power of each turbine. *Ans.* 648 h.p., 465 h.p.

17. The average flow of a stream is 3000 cu. ft. per second and the pondage is very limited. The normal head is 30 ft. but is at times as low as 18 ft. What type of turbine should be employed, how many units should there be, and at what speed will they run?

Ans. 4 units at 124 r.p.m. probably best.

18. The average flow of a stream is 3000 cu. ft. per second. The normal head is 30 ft. which is decreased somewhat in times of flood. The stream flow is fluctuating with long low water periods, but there is considerable storage. The load on the plant also varies considerably. What type of turbine should be used, how many units should there be, and at what speed should they run?

19. A turbine is required to carry a constant load of 800 h.p. under a head of 120 ft. There is considerable storage capacity and the stream has

COST OF TURBINES AND WATER POWER

133. General Considerations.—Since there are so many factors involved, it is rather difficult to establish definite laws by which the cost of a turbine may be accurately predicted. No attempt to do so will be made here, but a discussion of the factors involved and their affects will be given and the general range of prices stated. A few actual cases are cited as illustrations.

A stock turbine will cost much less than one that is built to order to fulfil certain specifications. This fact is illustrated by the comparison of two wheels of about the same size and speed. The specifications of the stock turbine were as follows: 550 h.p. at 600 r.p.m. under a head of 134 ft., 26-in. double discharge bronze runner, cast steel wicket gates, cast-iron split globe casing 5 ft. in diameter, and riveted steel draft tube. Weight about 11,500 lb. Price \$1750. The special turbine was as follows: 500 h.p. at 514 r.p.m. under a head of 138 ft., bronze runner, spiral case, riveted steel draft tube, connections to header, relief valve, and vertical type 5000 ft.-lb. Lombard governor. Price \$4000. The latter includes a governor, relief valve, and some connections which the former did not; but the difference in cost is more than the price of these.

The cost of the turbine is also affected by the quality and quantity of material entering into it, the grade of workmanship, and the general excellence of the design. With the \$4000 turbine cited in the preceding paragraph another may be compared which is of superior design. The specifications for the latter were as follows: 550 h.p. at 600 r.p.m. under 142-ft. head, single discharge bronze runner, spiral case with 30-in. intake, cast steel wicket gates, bronze bushed guide vane bearings, riveted steel draft tube, lignum vitæ thrust bearing, oil pressure governor sensitive to 0.5 per cent. The guaranteed efficiencies were

83 per cent. at 410 h.p.

is evidenced by the bid of another firm, as follows: 550 h.p. at 600 r.p.m. under 142-ft. head, single discharge cast iron runner, spiral case, cast steel guide vanes, cast steel flywheel, oil pressure governor, connections to header, 30-in. hand-operated gate valve, riveted steel draft tube, and relief valve. The guaranteed efficiencies were

	Per cent. of max. h.p.
81.5 per cent. at	100
84.5 per cent. at	90
84.5 per cent. at	85
82.5 per cent. at	75
79.5 per cent. at	60

Weight of turbine complete 38,000 lb. Price $8740. This last turbine includes a few items that the former does not, but the difference in cost cannot be accounted for by them. It will be noted that a flywheel was deemed necessary here, while it was not used on any of the others. Compare the weights and costs of these last two turbines with the weight and cost of the stock turbine first mentioned.

134. Cost of Turbines.—The cost of a turbine depends upon its size and not upon its power. Since the power varies with the head, it is apparent that the cost per h.p. is less as the head increases. Thus a certain 16-in. turbine (weight = 7000 lb.) without governor or any connections may be had for $1000. Under various heads the cost per horsepower would be as follows:

Head	H.p.	Cost per h.p.
30 ft.	52	$19.20
60 ft.	148	6.75

power might be only $0.005. Under a 20-ft. head a much lighter and cheaper construction would be entirely reasonable, while under a 300-ft. head the turbine would have to be built stronger and better than this one was.

For a given head, the greater the power of the turbine the less the cost per horsepower will be. Also for a given head and power, the higher the speed, the smaller the wheel, and consequently the less the cost. Compare the 600-r.p.m. reaction turbines in Art. 133 with the following, which is a double overhung tangential water wheel at 120 r.p.m. The horsepower is 500 under 134-ft. head. Oil pressure governor is included, but no connections to penstock are furnished. Weight 80,000 lb. Price $8900.

These last differences are very much magnified if we combine the cost of the generator with that of the turbine. The following are some generator quotations. The first is that of a generator at a special speed. The second is that of a generator of somewhat better construction than the first but of a standard speed. The others are all standard speeds.

150 kv.-a., 2400 volts, 3-phase, 60-cycle, 124 r.p.m.	$4850.
150 kv.-a., 2400 volts, 3-phase, 60-cycle, 120 r.p.m. (Weight 17,210 lb.)	$3300.
300 kv.-a., 2400 volts, 3-phase, 60-cycle, 120 r.p.m. (Weight 25,520 lb.)	$4700.
350 kv.-a., 2400 volts, 3-phase, 60-cycle, 514 r.p.m.	$2330.
350 kv.-a., 2400 volts, 3-phase, 60-cycle, 600 r.p.m.	$2100.

Taking the highest priced 600-r.p.m. turbine and combining it with the 350-kv.-a. generator we get a total of $10,850. Adding the cost of the 120-r.p.m. turbine to that of the 300-kv.-a. generator we get a total of $13,600 for a smaller amount of power.

Prof. F. J. Seery has derived the following empirical formula based upon the list prices of 35 wheels made by 20 manufacturers.

Log $X = A + D/B$, in which X is the cost in dollars for a

$X = 0.045\ D^{2.25}$, in which X is in dollars and D is the diameter of the runners in inches.

The cost of the casing increases these values very greatly, as some spiral cases may cost much more than the runner. A single case may be cited of a pair of 20-in. stock runners in a cylinder case with about 30 ft. of 5 ft. steel penstock. Each runner discharges into a separate draft tube about 3 ft. long. The power is 150 h.p. under 30-ft. head. The cost was \$2000.

A few quotations are here given. A reaction turbine to develop 4000 h.p. at 600 r.p.m. under 375 ft. head and weighing 90,000 lb. would cost \$14,000. Another reaction turbine of 10,000 h.p. under 565-ft. head cost \$37,000. In the latter case the governor, pressure regulator, and the generator were included. The building, crane, transformer room, etc., cost \$20,000 for this installation. A tangential water wheel of 2500 h.p. under 1200-ft. head cost \$12,000, while another of 4500 h.p. under 1700-ft. head cost \$8,000.

As has been stated, the cost of a turbine varies between fairly wide limits due to difference in design, workmanship, and commercial conditions. The cost per h.p. is also less the higher the head or the greater the power. In a general way it can be said to vary between \$2 and \$30 per horse-power and according to the following table:

Head	Cost per h.p.	Cost of building per h.p.
— -60 ft.	\$30–\$7	\$30–\$4
100–600 ft.	\$12–\$2	\$ 7–\$2
500–2000 ft.	\$ 8–\$2	\$ 7–\$2

The cost of the turbine is usually only about 6 per cent. of the total cost of the power plant. It scarcely pays, therefore, to buy a cheap turbine when the money saved is such a small portion of the entire investment.

125. Capital Cost of Water Power.—The capital cost of water-

the report of the Hydro-Electric Power Commission of the Province of Ontario. The proposed plant was to be located at Niagara Falls.

TABLE 9

Items	50,000 h.p.	100,000 h.p.
Tunnel tail race	$1,250,000	$1,250,000
Headworks and canal	450,000	450,000
Wheel pit	500,000	700,000
Power house	300,000	600,000
Hydraulic equipment	1,080,000	1,980,000
Electric equipment	760,000	1,400,000
Transformer station and equipment	350,000	700,000
Office building and machine shop	100,000	100,000
Miscellaneous	75,000	75,000
	$4,865,000	$7,255,000
Engineering, etc., 10 per cent	485,000	725,000
	$5,350,000	$7,980,000
Interest, 2 years at 4 per cent	436,560	651,168
Total capital cost	$5,786,560	$8,631,168
Capital cost per horsepower	$114	$86

The cost per unit capacity is usually less as the head increases. This is illustrated by the following table taken from Mead's "Water Power Engineering."

Capacity horsepower	Head	Capital cost per h.p.			
		Without dam	With dam	With dam and electrical equipment	With dam, electric equipment, and transmission line
8000	18	$63.50	86	115	150
8000	80	21.00	39	60	90

power will be the sum of the fixed charges and the operating expenses. The former will cover interest on the capital cost, taxes, insurance, depreciation, and any other items that are constant. The latter includes repairs, supplies, labor, and any other items that vary according to the load the plant carries. The annual cost per horsepower is the total annual cost divided by the horsepower capacity of the plant.

The total annual cost will vary with the number of hours the plant is in service and also with the load carried. The cost will be a maximum when the plant carries full load 24 hours per day and 365 days per year. It will be a minimum when the plant is shut down the entire year, being then only the fixed charges. (See

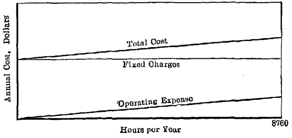


FIG. 119.

Fig. 119). It is evident that the annual cost per horsepower depends upon the conditions of operation.

However, under the usual conditions of operation, the annual cost may be said to vary from \$10 to \$30 per horsepower.

137. Cost of Power per Horsepower-hour.—In order to have a true value of the cost of power it is necessary to consider both the load carried and the duration of the load. While the annual cost per horsepower will be a maximum when the plant carries full load continuously throughout the year, the cost per horsepower-hour will be a minimum. Thus suppose the annual

per maximum horsepower may still be \$17, but the annual cos
per average horsepower will be \$68. This atter divided by
4380 hours gives 1.55 cents per horsepower-hour. It is clear
then, that the cost of power per horsepower-hour depend
very greatly upon the load curve. It may range anywhere from

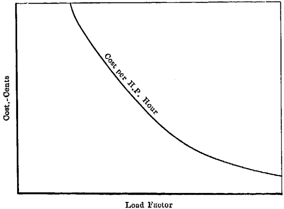


FIG. 120.

0.40 cents to 1.3 cents per horsepower-hour and more if the loa
factor is low. (See Fig. 120.)

138. Sale of Power.—If power is to be sold, one of the firs
requirements generally is that the output of the plant should b
continuous and uninterrupted. Such a plant should possess a
least one reserve unit so that at any time a turbine can be shu
down for examination or repair. This adds somewhat to th
cost of the plant. The larger the units the more the added cos
of this extra unit will be. On the other hand small units ar
undesirable since a large number of them make the plant to
complicated. Also the efficiency of the smaller wheels will b
less than that of the larger sizes. Unless the water supply i
fairly regular, storage reservoirs will be necessary and often auxi

margin of profit. The price for which the power may be sold is usually fixed by the cost of its production in other ways. This point should be carefully investigated and, if the cost from other sources is less than the cost of the water power plus the profit, the proposition should be abandoned.

139. Comparison with Steam Power.—It is necessary to be able to estimate the cost of other sources of power in order to tell

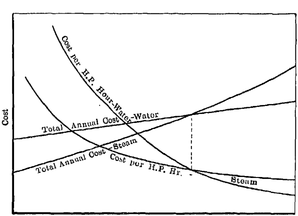


FIG. 121.—Comparison of costs of steam and water power.

whether a water-power plant will pay or not. Also it is often essential to figure on the cost of auxiliary power. As steam is the most common source of power and is typical of all others, our discussion will be confined to it.

In general the capital cost of a steam plant is less than that of a water-power plant. It varies from \$40 to \$100 per horsepower, with an average value of about \$60 per horsepower. Depreciation, repairs, and insurance are at a somewhat higher rate but, nevertheless, the fixed charges are less than for water power.

The amount of labor necessary is greater and this, together

state without a careful investigation. But it is clear that, as a rule, the cost of steam power is less when the plant is operated but a portion of the year or when the load factor is low. Thus a water-power plant is of the most value when operated at high load factor throughout the year.

The annual cost of steam power per horsepower is very high for small plants but for capacities above 500 h.p. it does not vary so widely. Its value depends upon the capacity of the plant, the load factor, and the length of time the plant is operated. It may be anywhere from \$20 to \$70, though these are by no means absolute limits.

Since the operating expenses are of secondary importance in a water-power plant, the annual cost per horsepower will not be radically different for different conditions of operation. But with a steam plant the annual cost per horsepower varies widely for different conditions of operation on account of the greater effect of the variable expenses. It is much better to reduce all costs to cents per horsepower hour. The accompanying table gives the usual values of the separate items that make up the cost of steam power, reduced to cents per horsepower hour.

Items	Min., cents	Max., cents
Fuel	0.20	0.75
Supplies	0.03	0.06
Labor	0.07	0.14
Administration	0.02	0.15
Repairs	0.05	0.10
Fixed charges	0.30	0.45
Total cost per horsepower hour	0.67	1.65

The following comparison is made by C. T. Main in *Trans.*

Repairs	2.0
Insurance	2.0
Total	12.5

For a steam plant at that location the capital cost was as $65 per horsepower. The annual cost per horsepower follows:

Fixed charges 12.5 per cent	$8.13
Fuel	8.71
Labor	4.16
Supplies	0.80
Total annual cost per horsepower	$21.80

For a water plant the cost of the power house and equi was taken as $65 while the cost of dams and canals at tha averaged $65 also, making a total capital cost of $130 per power. The annual cost per horsepower was as follows:

Fixed charges 9 per cent	$11.70
Labor and supplies	2.00
Total annual cost per horsepower	$ 13.70

However, for the case in question, a steam-heating pla necessary and its cost was divided by the horsepower plant giving the capital cost of the auxiliary steam plant a per horsepower of the power plant. The cost of its op based upon the power plant would be,

Fixed charges at 12.5 per cent	$0.94
Coal	3.26
Labor	1.23
Total cost of heating per horsepower of plant	$5.43

cost of steam power will be a minimum and it may be impossible for water power to compete with it. However where the cost of fuel is high water power may be a paying proposition even though its cost may be relatively high.

140. Value of Water Power.—The value of a water power is somewhat difficult to establish as it depends upon the point of view. However, the following statements seem reasonable:

An undeveloped water power is worth nothing if the power, when developed, is not more economical than steam or other power. If the power, when developed, can be produced cheaper than other power, then the value of the water rights would be a sum the interest on which would equal the total annual saving due to the use of the latter. Thus, referring to the case of Mr. Main cited in the preceding article, suppose the water supply is capable of developing 10,000 h.p. The annual saving then due to its use would be \$26,700 as compared with steam. Its value is then evidently a sum the interest on which would be \$26,700 per year.

A power that is already developed must be considered on a different basis. If the power cannot be produced cheaper than that from any other available source, the value of the plant is merely its first cost less depreciation, or from another point of view the sum which would erect another plant, such as a steam power plant, of equal capacity. If the water power can be produced cheaper than any other, the value of the plant will be its first cost less depreciation added to the value of the water right as given in the preceding paragraph.

141. QUESTIONS AND PROBLEMS

1. What are the general factors that affect the cost of a turbine of a given speed and power.?
2. What factors affect the cost of a turbine per h.p.?
3. What is meant by capital cost of water power? What items does it include? How is this cost per h.p. affected by the head and by the size of the plant?

6. How do water and steam power compare in general as to capital cost per h.p. and hence as to fixed charges? How do they compare as to operating expenses. How do the total annual costs and the cost per h.p. hour vary for each as functions of load factor?

7. How is it to be determined beforehand whether a water power plant will pay or not?

8. How is the value of a water right to be determined?

9. How is the value of an existing water power plant to be computed? Can there be any doubt about the correctness of the method?

10. Suppose you were called upon to make a report upon a water-power development, the only information given being the head available and the location for the plant, together with an assurance of a market for all power produced. How would you determine: (*a*) Amount of power that can be developed; (*b*) How much storage capacity should be provided; (*c*) Whether the plant should be built at all; (*d*) Value of the water right; (*e*) Size of penstock to be used; (*f*) Type of turbine to be used; (*g*) Number, size and speeds of units to be used?

11. If steam power costs $20 per h.p. per year and water power can be produced for $19 per h.p. per year, what would be the value of an undeveloped water right of 5000 h.p?

12. A water power plant cost $100 per h.p. and is estimated to have depreciated 15 per cent. If it costs $20 per h.p. per year to produce power from it in a place where steam power would cost $23 per h.p. per year, what is the value of the development?

142. General Dimensions.[1]—Assume that the head, speed, and power for a proposed water wheel are known, these values being so selected as to give the specific speed necessary for the type of impulse wheel desired. It is to be understood that the head is that at the base of the nozzle, and the power is the output corresponding to one jet. The velocity of the jet is given by the equation, $V_1 = c_v\sqrt{2gh}$, where the value of the velocity coefficient may be taken as 0.98. (See Fig. 89, page 114.) Since $B.h.p. = qhe/8.8$, we may write

$$q = \frac{8.8 \times B.h.p.}{he} = \frac{\pi d^2}{4 \times 144} c_v \sqrt{2gh} \qquad (60)$$

where d is the diameter of the jet in inches. From this the value of d may be found to be

$$d = 14.33\sqrt{\frac{B.h.p.}{e \times h^{3/2}}} \qquad (61)$$

The diameter of the wheel may be found from equation (47), which gives

$$D = \frac{1840\ \phi_e\sqrt{h}}{N}$$

where D is the diameter in inches of the "impulse circle," which is the circle tangent to the center line of the jet. The overall diameter of the runner depends upon the dimensions of the buckets. The value of ϕ_e is from 0.43 to 0.47.

143. Nozzle Design.—The nozzle tip and needle should be so proportioned as to give a constantly decreasing stream area from a point within the nozzle to a point in the jet beyond the tip of the needle, so that the water may be continuously accelerated. This must be so for every position of the needle. The curve of

[1] In this book only the hydraulic features of design will be considered. No space will be devoted to the determination of dimensions which can be

rate of discharge vary approximately in direct proportion to the linear movement of the needle. (See Fig. 89.)

The diameter of the orifice of the nozzle tip must be greater than the diameter of the jet, due to the contraction of the latter and also to the space taken up by the needle tip, which is never entirely withdrawn. At wide open setting the needle tip may

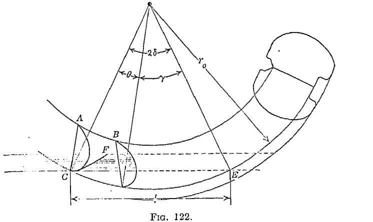


FIG. 122.

occupy 10 per cent. or more of the area of the orifice. The remaining area, through which the water passes, may be computed from the *area* of the jet by the use of a coefficient of contraction, typical values for which are given in Fig. 89, page 114. In reality the effective area of the nozzle is that perpendicular to the stream lines and is the surface of the frustum of a cone, whose elements are perpendiculars dropped from the edge of the orifice to the needle. This area is slightly greater than that in the plane of the orifice. The nozzle tip diameter shoud be computed for a size of jet sufficiently large to carry the maximum load on the wheel.

144. Pitch of Buckets.—In Fig. 122 the bucket A has just com-

$$\theta = 2\delta - 2\frac{u_0}{V_1}\sin\delta \qquad (62)$$

But this value of the pitch angle would be such as to permit the particle of water to merely touch the bucket before the latter swung up out of its line of action. In order to permit the water to flow over the bucket a closer spacing than this is required. The time necessary for flow over the bucket may be represented by $t' = l'/v'$, where l' is the length of path and v' the velocity relative to the bucket, a mean value being chosen between v_1

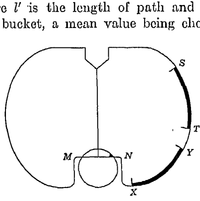


FIG. 123.

and v_2. It appears rather difficult to express this readily in a simple formula and the practical procedure appears to be to assume an approximate spacing for the buckets and then compute the probable time required for a particle of water to complete its flow. As a preliminary trial value we may assume the above value to be reduced by 20 per cent., in which case the number of buckets n may be found by

$$n = 2\pi/0.8 \times \theta \qquad (63)$$

As noted above, this value should be checked by a numerical computation.

So far the bucket has been considered as if all points on its lower edge were at the same distance from the axis, whereas the buckets of the present day have...

E in Fig. 122 to fully act. In other words, referring to Fig. 122, the entrance edge of the bucket describes the arc CE, but the extreme discharge edge describes a larger arc. This permits the use of fewer buckets on a wheel without involving any loss. It is not desirable to extend the part MN to the same radius because that would make conditions less favorable when the bucket first enters the jet.

For a high specific speed wheel the runner diameter becomes relatively smaller for the same jet diameter and this shortens the length of the path CE (Fig. 122). In order that all the water may be fully utilized it is necessary to reduce the time required for the particle of water at C to catch up with bucket B. This can be done by reducing the pitch.

But there is evidently a limit to this for mechanical reasons, as a certain amount of metal is necessary in order that each bucket may be securely fastened to the rim. Furthermore, the closer the buckets are placed together the quicker must the water discharged be gotten out of the way of the following bucket. This means that it must leave with a higher residual velocity, which means in turn that the kinetic energy lost at discharge is greater. This is one reason why the efficiency of an impulse wheel is less, if the specific speed is too high. After the number of buckets on a given wheel has been made a maximum, the only other means of increasing the specific speed is to lengthen the buckets still more, but this evidently soon reaches its limit.

The curve representing the end of the portion of the jet intercepted by the bucket A may be drawn by plotting the path of the tip of the bucket relative to the jet. By computing the time necessay for bucket B to get to its extreme right hand position and then by moving CF the distance the water would travel in the same time interval, it is apparent whether any water is not utilized or not, and also the amount wasted can be approximately determined.[1]

[1]See, "Theory of the Tangential Waterwheel," by R. L. Daugherty in

and naturally ...

In order to get the proper bucket shape, curves may be plotted showing the path of the jet relative to the wheel, as in Fig. 124. In the figure only one such curve is shown, that for the top of the jet, and also we consider here only one section, that in the plane of the paper; but other stream lines and other parallel planes should also be used. It should be noted that this is the relative path for the free jet only. As soon as the water flows over the buckets its absolute velocity is altered, and consequently its

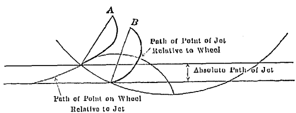


FIG. 124.

relative velocity and path are different. But as the function of these curves is merely to aid in determining the entrance conditions they are sufficient. As the bucket first enters the jet, the water flows in over the lip in the center of the notch, as MN in Fig. 123. It is only after the bucket has travelled somewhat farther that the water strikes it fully on the "splitter." As seen in Fig. 124, the face of the bucket along the lip should be such that the surface is approximately tangent to the relative path of the water, in order not to have any loss of energy at this point. After the bucket has moved along to another position where the jet strikes it in another place, the shape of that portion will be determined in the same way, but of course another portion of the curve will be used. Also the "splitter" should be approximately perpendicular to the relative path. (See Fig. 23.) It should be borne in mind that where the relative and absolute

[1] See paper by Eckart to which reference is made in note on page 145,

actual position of the particle of water as it leaves the right hand side of the wheel in its absolute path. The use of these curves will enable one to determine the best shape for the bucket along the lip and along the splitter, as well as the best outline for the notch.

But of equal importance with the design of the face of the bucket is that of the shape of the back. As the bucket A enters the jet in Fig. 124, its back should not intersect the curve of the relative path of the water. If it does intersect it, it indicates that the back of the bucket will strike the water in the jet and it is obvious that this would result in a loss of efficiency. The back of the bucket could strike the water, despite the higher velocity of the latter, because they are not moving in the same direction. Hence the back of the bucket should be no more than tangent to the curve shown. It is obvious that this matter should be investigated for other stream lines and other planes, besides the one shown.

Ideally the water should be reversed by the bucket and discharged backwards, relatively, at an angle of 180°. But this is impractical because the water would then be unable to get out of the way of the next bucket. Hence such an angle should be used as will enable the water to be discharged with an absolute velocity whose lateral component is sufficient. As has been pointed out, the closer the buckets are placed, the greater must be the value of this velocity and hence the more this angle must be made to differ from 180°. The bucket angle used in practice is about 170°.

If the shape of the bucket can be determined for the entrance and discharge edges by the application of the preceding principles, the bucket may be completed by joining these two portions with any smooth surface of double curvature. There should be no sharp curvature used nor anything which would tend to cause any abrupt change in the path or velocity of the water.

146. Dimensions of Case.—The case should be made of suffi-

otherwise water will be thrown back upon the wheel and thus increase the so-called windage loss. This action is most marked in many of the small laboratory wheels that have been made with very narrow cases.

147. QUESTIONS AND PROBLEMS

1. How may the diameter of a Pelton wheel be found for a given head, speed, and power? How may the diameter of jet be found?

2. What are the principles in the design of a needle nozzle? How may the size of the nozzle tip be determined, if the jet diameter is given?

3. How may the necessary pitch for the buckets of an impulse wheel be computed?

4. Why is the tangential water wheel bucket made as it is with a notch in the edge? Would it be possible to have an efficient bucket without this?

5. How may the specific speed of a Pelton wheel be increased? What limits the maximum value of the specific speed?

6. How may the shape of the bucket at the entrance edge be determined? How is the shape of the entire bucket fixed?

7. Suppose an impulse wheel is required to deliver 5,000 h.p. at 300 r.p.m. under a head of 1200 ft. Find the diameter of jet and the diameter of wheel necessary.

8. What would be the approximate diameter of the orifice of the nozzle tip in problem (7)?

9. What would be the probable pitch of the buckets in problem (7) and how many of them would be used on the wheel?

10. The bucket for the wheel in problem (7) may be laid out on the drafting board.

148. Introductory.—Assume that the head, speed, and power for a proposed turbine are known, the speed and power of the runner having been so chosen as to give the specific speed necessary for the type of turbine desired. The type of runner will have been selected in accordance with the principles and considerations of the preceding chapters, so that, as the problem comes to the designer, it is merely a matter of designing a turbine to fit the specified conditions.

It has been seen that practically all dimensions, factors, and even characteristics can be expressed as functions of the specific speed, hence the latter is the logical key to design. After the specific speed of the desired unit is known, the proper factors may be selected in the light of previous experience, and the necessary dimensions computed.

The data given in this chapter is to be understood as merely typical of present practice. It is perfectly possible to alter any of the quantities given, within certain limits, providing other related factors are changed also. Consequently runners of the same specific speed may be built without their being identical in all other respects. Also, of the numerous variables, certain ones are assumed and the rest computed to correspond. It is apparent that the practice of designers may vary according to what is assumed and what is computed, and hence the procedure given here is not the only one that may be followed.

149. General Dimensions.—As explained in Art. 37, the value of ϕ_e increases in rational design as the specific speed increases. Customary values of this factor for different values of N_s are given by a curve in Fig. 126. As stated in the preceding article, this curve is not intended to be followed precisely, but the variation from it should not be too great.

Having selected a suitable value of ϕ_e, for the type of runner desired, the diameter may be computed from equation (52), which reduces to

$$c_r = \frac{6.01q}{BD\sqrt{h}} \tag{65}$$

$$c_r = \frac{0.0000157N_s^2}{\phi_e^2(B/D)e} \tag{66}$$

As an illustration of the possible variation in procedure, it may be noted that we compute c_r after assuming the value of B. It

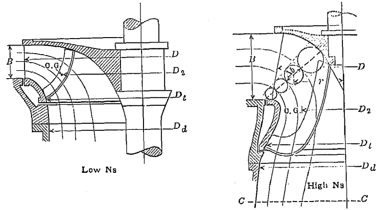


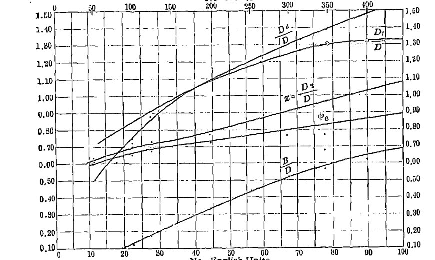
FIG. 125.

would be equally proper to assume a value of c_r and compute the corresponding value of B. It should also be noted that the above factors involve the assumption that 5 per cent. of the total area is taken up by the runner vanes. After the design has progressed to the point where the number and thickness of the vanes can be determined, the above may be corrected if that refinement is deemed necessary.

From equation (39), letting $c_u = c_e \cos \alpha_1$, we have

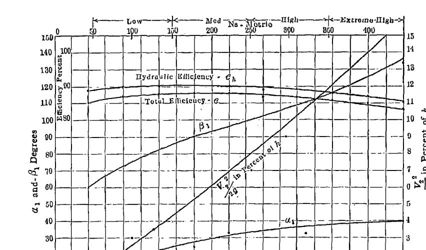
$$c_u = e_h/2\phi_e. \tag{67}$$

In the case of high specific speed runners this value needs to be increased about 5 per cent., but it is substantially correct for



D1/D
D2/D
x = D2/D
φe
B/D
Ns - English Units

Fig. 126.



Low
Med
High
Extreme-High
Ns - Metric
Hydraulic Efficiency - e_h
Total Efficiency - e
Efficiency Percent
β1
α1
α1 and β1 Degrees
V2²/2g in Percent of h

$$\tan \beta_1 = c_r/(c_u - \phi_e). \qquad (69)$$

The number of guide and runner vanes to be used is decided somewhat arbitrarily, but one fundamental principle to be observed is that they should not be equal to each other nor any simple multiple, otherwise pulsations will be set up. For simplicity of design and shop reasons it is convenient to make the guide vanes a multiple of 4. Zowski's rule is that the number of guide vanes, n', may be found by

$$n' = K'\sqrt{D} \qquad (70)$$

where $K' = 2.5$ for $\alpha_1 = 10°$ to $20°$, 3.0 for $\alpha_1 = 20°$ to $30°$, 3.5 for $\alpha_1 = 30°$ to $40°$. Although K' increases with the specific speed, the diameter of the runner decreases for the same power so that actually the number of vanes is often less.

In order to avoid any pulsations the runner vanes are often made an odd number, though other designers prefer to use an even number which is 2 less than the number of guide vanes. Zowski's rule is that the number of runner vanes, n, may be found by

$$n = K\sqrt{D} \qquad (71)$$

where $K = 3.7$ for a low specific speed, 3.0 for a medium specific speed, and 2.2 for a high specific speed.

150. Profile of Runner.—The profile of a runner is shown in Fig. 125 and the notation applied to it is clearly indicated. By D_2 is meant the diameter of the circle passing through the center of gravity of the outflow area. In Fig. 34 were shown a few typical profiles, and a more complete set is shown in Fig. 128.

The exact shape of profile desired is determined largely by experience, a shape being used that had been found satisfactory for the specific speed in question. But it is also a matter of the whim or taste of the designer, as theory has little bearing on it directly. But the theory (Art. 66) does indicate that a very sharp radius of curvature is undesirable and an excessive curva-

though it is frequently a little more.

With the values given by the curves in Fig. 126 and the aid of the samples shown in Fig. 128 and elsewhere, it is possible to lay out a profile that should be satisfactory. But before drawing in the outflow edge, it is necessary to consider the stream lines. Let us assume that all particles of water flow with the

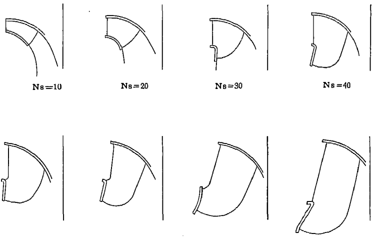


FIG. 128.—Typical profiles.

same velocities and that the total rate of discharge is divided into equal portions. If then the water passages are divided into portions of equal cross-section area, it follows that the boundary lines between them must be stream lines. Hence the height B at entrance may be divided into equal portions, and for our purpose here we shall assume four. Also the draft tube may be divided into concentric rings of equal area, as shown in Fig. 125, the section CC being removed far enough from the runner for the stream lines to have become parallel. Then the curves in between may be sketched in by eye. It may be noted that, if at an intermediate section a line be drawn normal to these stream

about 30 per cent. below the mean and that near the band about 50 per cent. above the mean velocity. This lowers the flow lines at entrance below the positions as determined above. Also the water in the draft tube, as in any other pipe, tends to flow with a higher velocity in the center than around the circumference. In accordance with these considerations the tentative flow lines, as first sketched in, may be modified, according to judgement. If further refinement is desired, this second set of flow lines may be checked and corrected by the method given in Appendix *B*.

The outflow edge may now be drawn by making it perpendicular to these various stream lines. But for the portion near the crown this procedure may tend to bring the discharge edge too close to the axis of rotation. The theory (Art. 91) indicates that a large variation in the radii to points along the outflow edge is undesirable, and that the discharge edge should really be parallel to the axis of rotation. The latter is not practicable, but for this portion of the outflow edge a compromise is effected by making it about a mean between a line parallel to the axis and one which would be normal to all the stream lines.

It should be noted that in these profiles we are dealing with circular projection, by which is meant that points are rotated about the axis of the runner until they lie in the plane of the paper. Thus the actual stream lines are not as shown in such a view, the lines drawn being merely circular projections of the actual paths.

151. Outflow Conditions and Clear Opening.—In case a stream line is not perpendicular to the outflow edge it indicates that the relative velocity of the water leaving the runner is not really normal to the outflow area, as the latter would ordinarily be measured. It is thus more convenient to deal with components of the velocity in a plane normal to the discharge edge at the point in question. Referring to Fig. 129, let us assume that the outflow edge is actually in the plane of the paper. If $\alpha_2 = 90°$ be assumed to be the conditions for which the runner is to be designed, the absolute velocities of the water at all points along

taken up by the vanes, hence the V_2 at discharge from the runner should be larger than that in the tube by a factor which may be assumed to be about 15 per cent. The absolute velocity of the water is ordinarily assumed to be uniform all along the outflow edge, but actually there may be some variation in it in certain types of runner because of the varying radii of curvature of the different stream lines.

Values of V_2 may be laid off along their respective stream lines as indicated in Fig. 129, and, where they are not perpendicular to the stream line, components indicated by V_2' should be found. The latter will be used in the diagram below.

The linear velocity of a point on the runner may be laid off at any radius perpendicular to a line representing this radius and a third line then drawn so as to form a triangle. It is often convenient to lay off u_1 at radius r_1, as shown in Fig. 129. At any other radius the peripheral velocity is given by the intercept. Since $\alpha_2 = 90°$ and V_2 (or V_2') is known for each stream line, a velocity diagram may be drawn with u_2 as a base, and as many of these constructed as there are stream lines. It should be noted that the crown and band of the runner form boundaries and hence furnish stream lines also. The series of diagrams so constructed, as in Fig. 129, give the values of the relative velocities and the direction of the runner vane at outflow for various points along its edge, if the absolute velocity of discharge is to be at 90°.

The clear opening is the shortest distance from a point on the discharge edge of one vane to the back of the next vane. This is shown in Fig. 129 and it may be seen that the clear opening is practically equal to *pitch* $\times$ *sin* β'_2 − *vane thickness*. The pitch at any radius is known, since the number of runner vanes are known. It may conveniently be found by laying off a value for the pitch at some radius, similar to the procedure for the velocities above, and then the pitch at any other radius may be found by using the proper intercept. This diagram should be drawn

Referring to Fig. 129 again, it may be seen that the clear opening may be found graphically at any point by laying off a line from the end of the pitch distance perpendicular to the vector v'_2

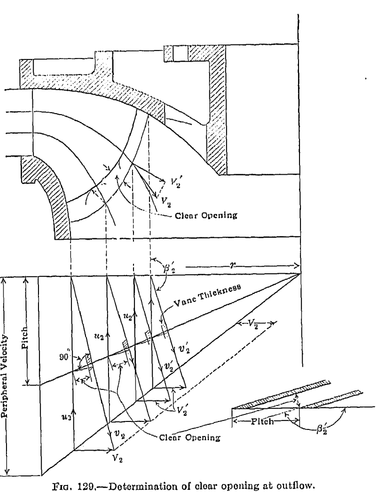


FIG. 129.—Determination of clear opening at outflow.

marked. Since the angle through which it is rotated is usually not very great, the error is small and hence this area in the plane of the paper may be taken as the true outflow area between two runner vanes.

The rate of discharge through any section of the outflow area may now be determined by multiplying each sectional area by the average value of the component of the relative velocity through it. In reality the true outflow area is normal to the true relative velocity, but in case the latter is not in the plane perpendicular to outflow edge, we obtain the same product by the method used.

The rate of discharge for the turbine may now be computed as

$$q = c\, n\, \Sigma v'_2 \Delta a_2 \qquad (72)$$

where n denotes the number of vanes, v'_2 the average value of the relative velocity for the section considered and in the plane defined, and Δa_2 the element of area between two stream lines. The coefficient of discharge may be taken as 0.95 for low specific speed runners, 0.90 for medium specific speed runners, and 0.85 for high specific speed runners. This is really a coefficient of contraction, since the actual stream areas are less than the areas at the end of the *converging* passages.

If the rate of discharge is not the exact quantity required, the outflow area may be altered somewhat by shifting the position of the crown or by changing the outflow edge until the desired result is obtained. It may be noted that the friction in flow through the runner passages will be less along the middle stream lines than for those near the crown and band. The relative velocity will thus be a little higher in the middle and to preserve a "radial" discharge all along the edge it will be necessary to decrease the angle β_2, which can be done by increasing the clear opening a trifle along the middle portion. By the same line of reasoning the opening may be reduced slightly at each end.

152. Layout of Vane on Developed Cones.—By the methods

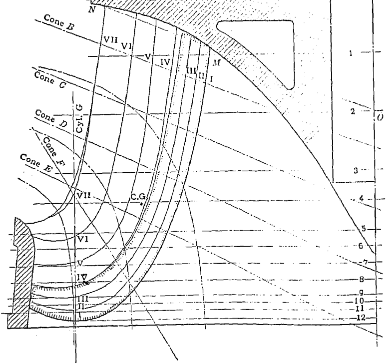


FIG. 130.

These elements should be so taken as to cut both the discharge and the entrance edges and it is desirable also to have them approximately perpendicular to the outflow edge. If this requirement in some instances causes the vertex of a cone to be removed to too great a distance, a cylinder may be used instead, the cylinder being a special case of the cone.

It is desirable to begin with the cone nearest the crown, as

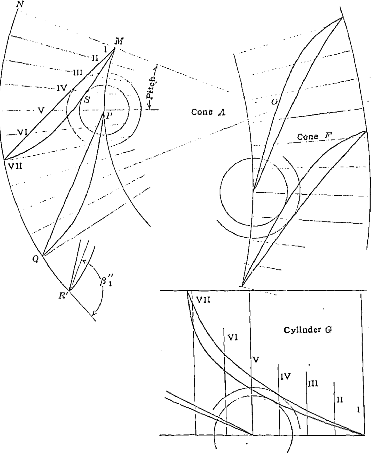


FIG. 131.—Layout of vanes on developed cones.

From P are described two arcs whose radii are equal to the clear opening at M and the clear opening plus the vane thickness. From M the two sides of a vane are then drawn tangent to these arcs, as shown. The actual vane is sharpened on the end, so as

the shape of discharge ends of the vanes are completely fixed. It may be noted that the clear opening PS is a constant distance for any cone taken through point M in Fig. 130.

In the case of the entrance ends of the vanes we might proceed in the same way, but it is usually more convenient to lay off the vane angle instead. It may be noted however that the value of

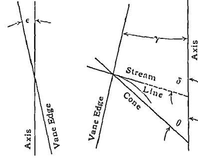


FIG. 132.

the angle on the developed cone is different for different cones through the same point. In the most general case the entrance edge of a runner may be inclined to the axis of rotation at an angle γ as shown in Fig. 132, and also it may not be in the same plane as the axis but in another plane at an angle ϵ. Let the elements of the cone make an angle θ with the axis, while the projected stream line at entrance makes an angle δ with the axis. It may be noted that the actual velocity diagram should always be in the plane of the stream line, and is not necessarily in a plane perpendicular to the axis. If the angle of the relative velocity β_1 becomes β''_1 on the developed cone, it may be proven by geometry and trigonometry that

$$\tan \beta''_1 = \tan \beta_1 \frac{\sin (\delta + \gamma)}{\sin (\theta + \gamma)}$$

$$\left[\frac{1}{\sin (\theta - \delta) \cdots} \right] \qquad (72)$$

smooth-curve may join this end of the vane with the portion MS. The complete vane is now MR. The next vane PQ should be drawn in order to find out if the cross-section area continuously decreases from entrance to outflow.

A similar procedure may be gone through with for the other cones or cylinders, except for one restriction. The relative position of M and R is purely arbitrary in the first cone used, but for all the remainder this much is fixed. Usually runners are so constructed that all points along the outflow edge are in one plane and furthermore this plane contains the axis of rotation. Our discussion will therefore be confined to this case, though the method could readily be extended to the more general treatment, if desired. If all points along the entrance edge are in the same plane and this plane contains the axis (the angle ϵ being zero), the arc NR subtends an equal number of pitches or fractions thereof in every cone or cylinder. This is most conveniently laid off on the drawing board by establishing an arc at a fixed radius from the axis of rotation and putting this in every cone. Its length is the same in every case. It is convenient to take this arc through the point where the diameter D is measured.

If the entrance edge is inclined, as indicated by the angle ϵ in Fig. 132, the runner vane near the crown subtends a greater angle than that portion nearer the band, and a different length of arc is used in different cones.

If it is difficult or impossible to secure proper vane curves in some of the cones, it may be necessary to go back to cone A and to change the position of R so that the vane MR subtends a different angle. One advantage of inclining the entrance edge, so that it makes the angle as shown, is that it permits of securing better vane curves in all the cones, in some instances. This is particularly true in the case of high specific speed runners.

153. Intermediate Profiles.—The various cones and cylinders of Fig. 131 are next divided up into fractional pitches, preferably quarter pitches. These lines represent a series of planes passing

Since planes I, II, III, IV, etc. are the same in every cone, it follows that profiles can be drawn through all the points with the same number in Fig. 130. If the vane surface is proper,

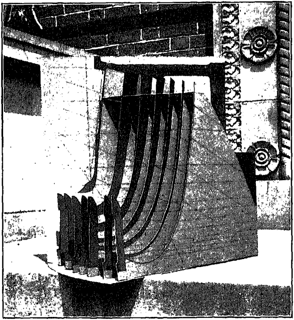


FIG. 133.—Model constructed by Lewis F. Moody illustrating development of vane surface.

these profiles will all be smooth curves, and will all be similar but changing gradually from the entrance edge to the discharge edge. If the curves are not smooth and of the proper shape, it will be necessary to change the vanes laid out on the developed cones until both the profiles and the curves on all the cones are satisfactory. Thus these profiles serve as a check on the work, and also are desirable in order to determine the pattern maker's sections.

curve in it which may be found as follows. In Fig. 134 a portion of this plane is drawn and it is subdivided into the same fractional pitches as the various developed cones. The distance XY may be transferred from Fig. 130 to Fig. 134 and laid off in plane I.

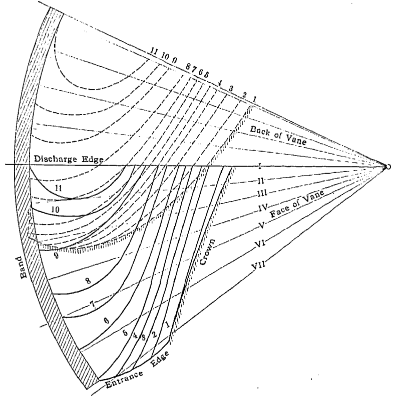
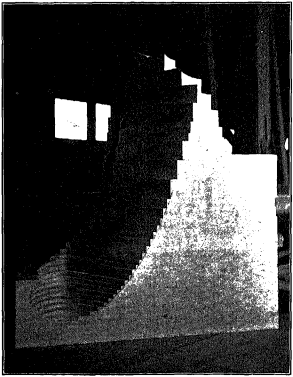


FIG. 134.—Patternmaker's sections.

The distance XZ may be transferred in similar manner and laid off along plane II. Proceeding in this way the entire curve may

Fig. 134 may be laid out. These boards, when placed together, as in Fig. 135, and the surface smoothed down, give the proper shape of the vane surface. In this way the core box for the vane may be formed.

Since the vane has both a front and a back surface which differ slightly from each other, this entire proceeding is carried through



(*Courtesy of Wellman-Seaver-Morgan Co.*)
FIG. 135.—Construction of pattern for rear face of core box for runner vane.

for both surfaces. It is desirable to draw the profiles and pattern maker's sections for the front of the vane in full lines and for the

shown in Fig. 136. If the cross-section of the case is circular, it may be seen that the radius of a point on the outer boundary is given by $r = \sqrt{c\theta} + K$ where c and K are constants and θ the subtended angle, as this curve will give an area which is directly proportional to the angle. For any other change of cross section, it is easy to determine the form necessary by applying the principle that the area must vary as the subtended angle at the runner axis.

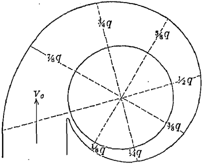


Fig. 136.

The path of a free stream line in the case may be plotted by the principles of Art. 66. If V_o be the velocity with which the water enters the case and at radius r_o, the tangential component of the velocity at any other radius is given by

$$V_u = r_o\, V_o/r$$

while the radial component is given by

$$V_r = q\,/2\pi rb$$

where r is the radius in feet to any point and b the height of the water passage in feet. These two components give the direction of the water at any radius, and by sketching in a series of tangents it is easy to plot the path by a little trial.

If speed ring vanes are used, they should be so shaped as to

that they are tangent to the free stream lines of the water entering. If the turbine is set in an open flume or case where free stream lines cannot readily be plotted, the guide vanes are made so as to approach a radial direction at this point.

The direction of the water is changed during flow through the guide passages from that of the free stream line to the direction desired. After a particle of water passes point a in Fig. 137, its direction should remain unchanged until it strikes the runner, as it is now following a free stream line once more. Since the space between guide vanes and runner vanes is one of a uniform height, it may be shown that free stream lines are then equi-angular or

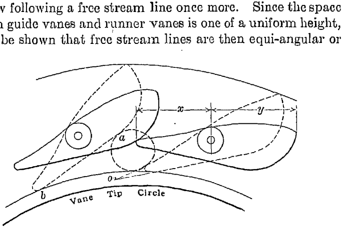


FIG. 137.

logarithmic spirals. Thus the portion of the vane from a to b should be such a curve. The other side of the vane may be either a straight line or another logarithmic spiral. The equation of the equi-angular spiral is $\log_e r = \theta \tan \alpha$, where α is the angle desired and θ is the subtended angle.

There should be considerable clearance between the ends of the guide vanes and the runner vanes so that the streams of water from the guides may unite into a solid ring before entering the runner. In particular the point of intersection o of Fig. 137 should be located outside the runner, so that no eddies may be

157. QUESTIONS AND PROBLEMS

1. Given the head, speed and power for a reaction turbine, how may the size of the runner, the height of the guide vanes, and the diameter of the draft tube be determined?

2. For the case in problem (1), how would the guide vane angle and the runner vane angle be determined? What principles are involved in deciding upon the number of guide vanes and runner vanes?

3. How is the profile of a runner to be fully determined? How should the stream lines be drawn in?

4. How may the clear opening of a turbine runner be determined?

5. How may the capacity of a runner be checked? What changes can be made in order that its capacity may be exactly that desired?

6. How are runner vanes laid out on developed cones?

7. Having the vanes laid out on developed cones, how may the intermediate profiles be constructed? What use is made of these?

8. How are pattern maker's sections drawn?

9. What is the object of plotting the free stream lines in a spiral case?

10. How should guide vanes be shaped? What other factors should be considered in their design?

11. A turbine runner is to deliver 4000 h.p. at 600 r.p.m. under a head of 305 ft. Determine D, B, D_t, D_d, α_1, β'_1, V_2, number of guide vanes, and number of runner vanes.

Ans. $D = 37$ in., $B = 6.67$ in., $D_t = 34.4$ in., $D_d = 33.3$ in., $\alpha_1 = 16°$, $\beta'_1 = 102°$, $V_2 = 25.6$ ft. per second, 24 guide vanes, and 22 runner vanes.

12. A turbine runner is to be designed for 2000 h.p. at 300 r.p.m. under a head of 88 ft. Find same as in problem (11).

Ans. $D = 43.2$ in., $B = 16.4$ in., $\alpha_1 = 20°$, $\beta'_1 = 125°$, 20 guide vanes, and 18 runner vanes.

13. A runner is to be designed to deliver 3000 h.p. at 200 r.p.m. under a head of 64 ft. Find the results called for in problem (11).

14. Find the allowable height above the tail water level for each of the runners in the preceding three problems.

15. Draw profile, sketch tentative flow lines, construct velocity diagrams, lay out vanes on cones, and draw patternmaker's sections for one of the turbines given above.

158. Definition.—Centrifugal pumps are so called because of the fact that centrifugal force or the variation of pressure due to rotation is an important factor in their operation. However, as will be shown later, there are other items which enter.

The centrifugal pump is closely allied to the reaction turbine and may be said to be a reversed turbine in many respects. Therefore it will be found that most of the general principles given in Chapter VII will apply here also with suitable modifications. Energy is now given up by the vanes of the impeller

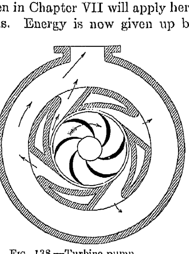


Fig. 138.—Turbine pump.

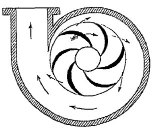


Fig. 139.—Volute pump.

to the water and we have to deal with a *lift* instead of a *fall*. The direction of flow through the impeller is radially outward. During this flow both the pressure and the velocity of the water are increased and when the water leaves the impeller a large part of its energy is kinetic. In any efficient pump it is necessary to conserve this kinetic energy and transform it into pressure.

159. Classification.—Centrifugal pumps are broadly divided into two classes:

1. Turbine Pumps.

chapter.

The turbine pump is one in which the impeller is surrounded by a diffusion ring containing diffusion vanes. These provide gradually enlarging passages whose function is to reduce the velocity of the water leaving the impeller and efficiently transform velocity head into pressure head. The casing surrounding the diffuser may be either circular as shown in Fig. 138 or it may be of a spiral form. This latter arrangement would be similar to that of the spiral case turbine shown in Fig. 55.

The volute pump is one which has no diffusion vanes, but, instead, the casing is of a spiral type so made as to gradually reduce the velocity of the water as it flows from the impeller to the discharge pipe. (See Fig. 139.) Thus the energy transformation is accomplished in a different way. The spiral curve for such a case is usually called the volute, and from this the pump receives its name.

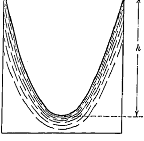


FIG. 140.

The discussion of the volute pump will apply equally well to all other types without diffusion vanes. The only difference will be that these other types are less efficient and also it will probably be impossible to express the shock loss at exit in any satisfactory way. Some of these other types have circular cases with the impeller placed either concentric or eccentric within them. Their only merit is cheapness.

160. Centrifugal Action.—If a vessel containing water or any liquid is rotated at a unifrom rate about its axis, the water will tend to rotate at the same speed and the surface will assume a curve as shown in Fig. 140. This curve can be shown to be a parabola such that $h = u_2{}^2/2g$, where u_2 = linear velocity of vessel at radius r_2. If the water be confined so that its surface cannot change, the pressure will follow the same law, as shown

as that for the turbine. To this, however, we shall add α'_2 as the angle the diffusion vanes make with u_2, and subscript (3) to denote a point in the casing.

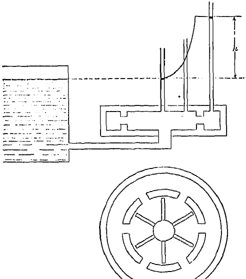


FIG. 141.

The actual lift of the pump will be denoted by h, while the head that is imparted to the water by the impeller will be denoted by h''. If h' represents all the hydraulic losses within the pump and e_h represents the hydraulic efficiency, we may write

$$h = e_h h'' = h'' - h' \tag{74}$$

It will also be found to be more convenient to express all velocities in terms of u_2 and v_2.

Whereas turbines are rated according to the diameter of the runner, centrifugal pumps are rated according to the diameter

pipes. It should be noted that the velocity head at the mouth of the discharge pipe is a discharge loss which should be added.

The head may also be obtained in a test by taking the difference between the total heads (Equation 3) on the suction and discharge sides of the pump. If the suffix (S) signifies a point in the suction pipe and suffix (D) a point in the discharge pipe we have

$$h = \frac{p_D}{w} - \frac{p_S}{w} + z_D - z_S + \frac{V_D{}^2}{2g} - \frac{V_S{}^2}{2g} \qquad (75)$$

In this case p_D/w represents the pressure gage reading reduced to feet of water while p_S/w represents the suction gage reading reduced to feet of water. In general the latter pressure will be less than that of the atmosphere. In such a case p_S/w will be negative in value.

The word efficiency without any qualification will always denote gross efficiency, that is the ratio of the power delivered in the water to the power necessary to run the pump. The hydraulic efficiency is the ratio of the power delivered in the water to the power necessary to run the pump after bearing friction, disk friction, and other mechanical losses are deducted. The hydraulic efficiency is therefore equal to Wh/Wh'' or h/h''. This latter expression is termed manometric efficiency by some and is treated as something essentially different from hydraulic efficiency. If the true value of h'' could be computed, the value of the hydraulic efficiency so obtained would be the same as that obtained experimentally by deducting mechanical losses from the power necessary to drive the pump. Actually the ratio of h/h'' will usually be less than this value but that is due to the fact that our theory is imperfect. (Art. 167.)

163. Head Imparted to Water.—By reversing equation (19) in Art. 60, since we are now dealing with a pump and not a turbine, we may write

$$h'' = \frac{h}{e_h} = \frac{1}{g}\,(u_2 V_{u_2} - u_1 V_{u_1}).$$

the eye of the impeller and even in the suction pipe may be set into rotation. The effect is as if the vanes of the impeller extended to this space. For this reason we shall drop the last term in the above equation and write

$$h'' = \frac{1}{g} u_2 V_{u_2} = \frac{u_2}{g} (u_2 + v_2 \cos \beta_2) \tag{76}$$

By another line of reasoning, or by a slight transformation of equation (76), we may obtain

$$h'' = \frac{u_2^2}{2g} - \frac{v_2^2}{2g} + \frac{V_2^2}{2g} \tag{77}$$

Sometimes one of these forms is more convenient than the other.

Inspection of equation (76) shows that if the pump is to do positive work, V_{u2} must be positive. Thus the absolute velocity of the water must be directed so as to have a component in the direction of rotation. If the pump speed, u_2, be assumed constant, equation (76) will plot as a straight line for values of v_2 (or q). If β_2 is less than 90°, the value of h'' will increase as the rate of discharge increases above zero. If β_2 is equal to 90°, h'' will be independent of the rate of discharge and will plot as a horizontal line for all values of v_2. If β_2 is greater than 90°, the value of h'' will decrease as the rate of discharge increases. (See Fig. 144.)

Since it is difficult to transform velocity head into pressure head without considerable loss, it is desirable to keep the absolute velocity of the water leaving the impeller as small as possible. For that reason the best pumps have vane angles as near 180° as possible in order that the relative velocity may be nearly opposite to the peripheral velocity of the impeller.

164. Losses.—In accordance with the usual methods in hydraulics, the friction loss in flow through the impeller may be represented by $kv_2^2/2g$, where k is an experimental constant. A study of Fig. 142 would indicate that there is no abrupt change of velocity at entrance to the impeller under any rate of flow;

case of the reaction turbine in Art. 86. Referring to Fig. 143, it

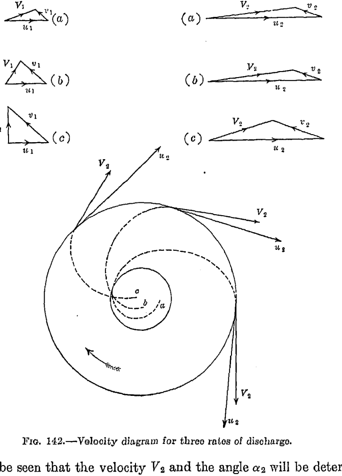


FIG. 142.—Velocity diagram for three rates of discharge.

may be seen that the velocity V_2 and the angle α_2 will be determined by the vectors u_2 and v_2. Since the vane angle α'_2 is fixed there can be only one value of the discharge that does not

$\sin \alpha'_2$

If $k' = \dfrac{\sin(\beta_2 - \alpha'_2)}{\sin \alpha'_2}$, then for the turbine pump the shock loss is approximately equal to

$$\frac{(u_2 - k'v_2)^2}{2g}\text{*}$$

For the turbine pump the total hydraulic loss may be represented by

$$h' = k\frac{v_2^2}{2g} + \frac{(u_2 - k'v_2)^2}{2g} \tag{78}$$

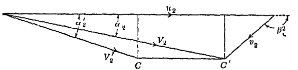


Fig. 143.

Since the volute pump has no diffusion vanes, there will be no abrupt change in the direction of the water at exit from the impeller, but there may be an abrupt change in the magnitude of the velocity. The water leaves the impeller with a velocity V_2 and enters the body of water in the case which is moving with a velocity V_3. In accordance with the usual law in hydraulics this shock loss may be represented by

$$\frac{(V_2 - V_3)^2}{2g}$$

For the usual type of pump V_2 will decrease as the discharge increases, and in any case V_3 must increase as the quantity of water becomes greater. If the discharge becomes such that the two are equal then there will be no shock loss. The value of V_2 may be expressed in terms of u_2 and v_2, and if the ratio of (a_2/A_3) be denoted by n, we have $V_3 = nv_2$. Making these substitutions

* L. M. Hoskins, "Hydraulics," p. 237.

Though the values of k may be different for the two types and though the expressions for shock loss are unlike in appearance, yet it can be seen that the losses in each case follow the same general kind of a law. In the turbine pump we have a gradual reduction of velocity but, except for one value of discharge, a sudden change in direction as the water leaves the impeller. With the volute pump we have no abrupt change of direction but a sudden change of velocity. The transformation of kinetic energy into pressure energy is incomplete in either case, but it is generally believed that the loss is somewhat greater in the volute pump than in the turbine pump.

For an infinitesimal discharge the value of the velocity in the case, V_3, would be practically zero. Therefore a particle of water leaving the impeller with a velocity V_2 and entering a body of water at rest would lose all its kinetic energy. For such a case, however, the value of v_2 would be also practically zero so that V_2 would equal u_2. Therefore for a very slight discharge the shock loss would be $h' = u_2^2/2g$. Such a value of h' may be obtained from either (78) or (79) by putting $v_2 = 0$.

165. Head of Impending Delivery.—The head developed by the pump when no flow occurs is called the shut-off head or the head of impending delivery. We are then concerned only with the centrifugal head or the height of a column of water sustained by centrifugal force. In Art. 160 this was shown to be equal to $u_2^2/2g$. The same result may be obtained from the principles of Art. 163 and Art. 164. If v_2 becomes zero, then by equation (76), $h'' = u_2^2/g = 2\,u_2^2/2g$. But, as was shown in Art. 164, the loss of head, $h' = u_2^2/2g$. Therefore $h = h'' - h' = u_2^2/2g$.

Although ideally the head of impending delivery equals $u_2^2/2g$, we find that various pumps give values either above or below that. This may be accounted for in a number of ways. In any pump we never have a real case of zero discharge; for a small amount of water, about 5 per cent. of the total rated capacity perhaps, will be short circuited through the clearance spaces. A

directed backward, the more tendency there is for internal eddies to be set up and these tend to decrease the head. Also if the water in the eye of the impeller is not set in rotation at the same speed as the impeller the head may be further reduced. There is also a tendency for the water surrounding the impeller to be set in rotation but this, on the other hand, helps to increase the head since the real value of r_2 is greater than the nominal value.

It will usually be found that actually the head of impending delivery may be from 0.9 to 1.1 $\frac{u_2^2}{2g}$.

166. Relation between Head, Speed and Discharge.—When flow occurs the above relation no longer holds, for other factors besides centrifugal force enter in. Due to conversion of velocity head into pressure head when water flows, a lift may be obtained which is greater than $u_2^2/2g$. (See Fig. 144.)

This may be shown best by equation (77), when the losses are introduced. The hydraulic friction loss in flow through the impeller may be represented by $kv_2^2/2g$. Then at discharge from the impeller a portion of the kinetic energy is lost within the diffuser or within the volute case, and the remainder may be represented by $mV_2^2/2g$, where m is a factor less than unity. Deducting the losses from the expression for h'' in equation (77) we have

$$h'' = \frac{u_2^2}{2g} - (1 + k)\frac{v_2^2}{2g} + m\frac{V_2^2}{2g} \qquad (80)$$

If the factor involving V_2 is greater than that with v_2 the head will be greater than the shut-off head, while the reverse is true if it is less.

In order to produce a pump with a rising characteristic, it is not only necessary to conserve the kinetic energy of the water discharged from the impeller, or in other words to keep the factor m high, but it is also necessary to have V_2 large and v_2 small. But a pump with a falling characteristic is not necessarily any less efficient than the former type. The factor m may be high but yet

th vane angles greater than 90,° and in fact as large as 154,°

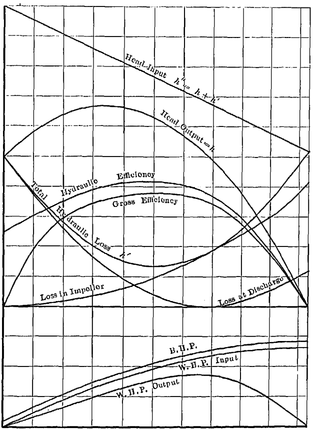


FIG. 144.—Ideal curves for a turbine pump.

y manifest decidedly rising characteristics, while certain impellers with radial vanes have given steep falling characteristics d not flat characteristics.[1]

While the above equation is very satisfactory in explaining

and also the rate of discharge affects two different variables v_2 and V_2, which are in reality related to each other. For some purposes, therefore, it is better to derive the following forms of equations.

The actual lift of the pump h may be obtained by subtracting the losses h' from the head h'' imparted by the impeller. The value of h'' will be taken as $\frac{u_2 (u_2 + v_2 \cos \beta_2)}{g}$ and the values of h' are given in equations (78) and (79).

Making these substitutions for the turbine pump we obtain after reduction

$$u_2^2 + 2 (k' + \cos \beta_2) u_2 v_2 - (k + k'^2) v_2^2 = 2gh. \qquad (81)$$

For the volute pump we obtain after rearranging

$$u_2^2 + 2nv_2\sqrt{u_2^2 + 2u_2v_2 \cos \beta_2 + v_2^2} - (1 + k + n^2) v_2^2 = 2gh \qquad (82)$$

These equations involve the relation between the three variables u_2, v_2, and h. Any one of these may be taken as constant and the curve for the other two plotted. If the pump is to run at various speeds under a constant head, the latter will then be fixed and we may determine the relation between speed and discharge. The more common case is for the pump to run at a constant speed. For that case values of h may be computed for different values of v_2. The curves for a turbine pump run at constant speed are shown in Fig. 144.

Although it will not be done here, it will be found convenient to introduce ratios or factors as was done in the case of the turbine. We may write $u_2 = \phi\sqrt{2gh}$ and $v_2 = c\sqrt{2gh}$ and using these in equations (81) and (82) we obtain relations between c and ϕ similar to equation (40). As in the case of the turbine it will be found that the best efficiency will be obtained for a certain value of ϕ and c. It will thus be clear that the speed of the pump should vary as the square root of the lift, and that the best value of the discharge will be proportional to the square

shape as those drawn for a constant speed it will be necessary to plot values of $\left(\frac{1}{\phi^2}\right)$ and of $\left(\frac{c}{\phi}\right)$.

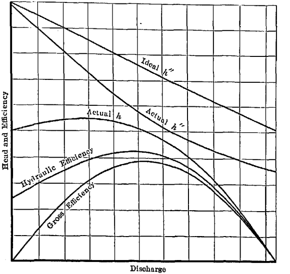


FIG. 145.—Actual curves for turbine pump.

The value of ϕ for the maximum efficiency depends upon the design of the pump. By choosing different values of β_2 and either α'_2 or n, and different numbers of impeller vanes and other factors, a pump may be given a rising or a flat or a steep falling characteristic. The values of ϕ for the highest efficiency range from about 1.30 down to about 0.90. This means that the normal head is usually

$$h = 0.6 \text{ to } 1.1\frac{u_2^2}{2g}.$$

The value of c_r, the coefficient of the radial velocity at the point of outflow from the impeller, is usually from 0.05 to 0.15. All formulas and values in this chapter are based upon the

and V_2, which are in reality related to each other. For som
purposes, therefore, it is better to derive the following form
of equations.

The actual lift of the pump h may be obtained by subtractin
the losses h' from the head h'' imparted by the impeller. Th
value of h'' will be taken as $\frac{u_2 (u_2 + v_2 \cos \beta_2)}{g}$ and the values of
are given in equations (78) and (79).

Making these substitutions for the turbine pump we obta
after reduction

$$u_2^2 + 2 (k' + \cos \beta_2) u_2 v_2 - (k + k'^2) v_2^2 = 2gh. \quad (8$$

For the volute pump we obtain after rearranging

$$u_2^2 + 2nv_2\sqrt{u_2^2 + 2u_2v_2 \cos \beta_2 + v_2^2} - (1 + k + n^2) v_2^2 = 2gh \ (8$$

These equations involve the relation between the three vari
ables u_2, v_2, and h. Any one of these may be taken as consta
and the curve for the other two plotted. If the pump is to ru
at various speeds under a constant head, the latter will then b
fixed and we may determine the relation between speed an
discharge. The more common case is for the pump to run at
constant speed. For that case values of h may be computed fo
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these in equations (81) and (82) we obtain relations between
and ϕ similar to equation (40). As in the case of the turbin
it will be found that the best efficiency will be obtained for
certain value of ϕ and c. It will thus be clear that the speed
the pump should vary as the square root of the lift, and that th
best value of the discharge will be proportional to the squa
root of the lift. Since $h = \frac{1}{\phi^2}\left(\frac{u_2^2}{2g}\right)$, it is apparent that the li

varies as the square of the speed. If this value of h be substituted in $v_2 = c\sqrt{2gh}$ we obtain $v_2 = \left(\frac{c}{\phi}\right) u_2$, and this shows that the best value of the discharge varies directly as the speed.

Curves between c and ϕ will be of the same appearance as those drawn for a constant value of h. To construct curves of the same shape as those drawn for a constant speed it will be necessary to plot values of $\left(\frac{1}{\phi^2}\right)$ and of $\left(\frac{c}{\phi}\right)$.

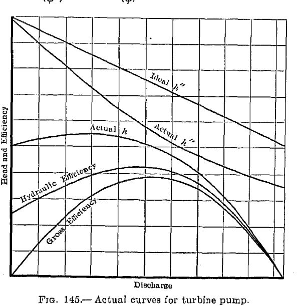

FIG. 145.—Actual curves for turbine pump.

The value of ϕ for the maximum efficiency depends upon the design of the pump. By choosing different values of β_2 and either v_2 or n, and different numbers of impeller vanes and other factors, a pump may be given a rising or a flat or a steep falling characteristic. The values of ϕ for the highest efficiency range from about 1.30 down to about 0.90. This means that the

sources of error lies in the assumptions made regarding losses. In particular the expressions for shock loss for either the turbine or the volute pump must be regarded as only rough approximations.

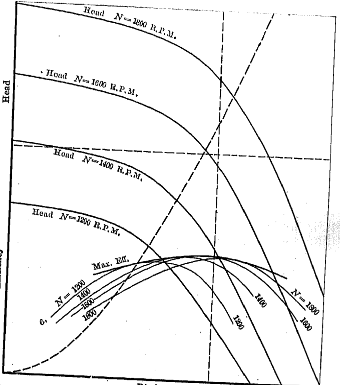

FIG. 146.—Curves for pump run at different speeds.

While actual tests have shown curves similar to the ideal curves given in Fig. 144, it is more common to find the relation between head and discharge to be like that in Fig. 145. In many cases also the pump has a falling characteristic

turbine. In addition there is strong reason for believing that there is a dead water space on the rear of each vane, thus the actual area a_2 will be less than the nominal area used in the computations. This is probably a larger item than the contraction of the streams mentioned in connection with the turbine. The ordinary pump has no guide vanes at entrance to the impeller and the conditions of flow at that point are uncertain.

The more vanes the impeller has the more perfectly the water is guided and the more nearly the actual curves approach the ideal. It is necessary to have enough vanes to guide the water fairly well but too many of them cause an excessive amount of hydraulic friction. Within reasonable limits—say 6 to 24—the efficiency is but little affected. If the use of few vanes lowers the value of h, the value of h'' is lowered at about the same rate so that the ratio of the two is but little altered.

168. Efficiency of a Given Pump.—If a given pump is run at different speeds the lift should vary as the square of the speed, the discharge as the speed, and the water h.p. as the cube of the speed. If the efficiency of the pump remained constant the horsepower necessary to run the pump would also vary as the cube of the speed. It is probable that the hydraulic efficiency is reasonably independent of the speed. The mechanical losses, however, do not vary as the cube of the speed. For low speeds the mechanical losses do not increase so fast and thus the gross efficiency of the pump will improve as it is used under higher heads at higher speeds. After a certain limit is reached, however, the mechanical losses follow another law and for very high speeds they will increase faster than the hydraulic losses and the efficiency will begin to decline. Thus for a given pump run at increasing speeds the maximum efficiency will increase and then decrease again. It is thus clear that the head which may be efficiently developed with a single stage is limited. For higher heads it is necessary to resort to multi-stages.

These conclusions regarding efficiency are borne out by the curves shown in Fig. 146 and Fig. 147. In the latter the highest

the efficiency curves for the various speeds.

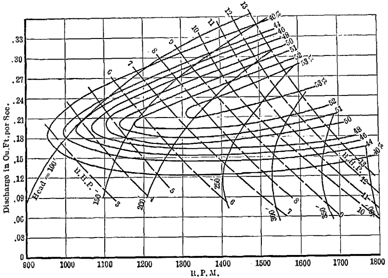

FIG. 147.—Characteristic curve of a 4-stage turbine pump.

169. Efficiency of Series of Pumps.—For a given pump the speed and head are seen to have some influence upon its efficiency. However, the capacity for which it is designed is the greatest factor. Suppose we have a series of impellers of the same diameter and same angles running at the same speed, the lift will be approximately the same for all of them. Suppose, however, that the impellers are of different widths. The discharge will then be proportional to the width and the water horsepower is proportional to the discharge. But the bearing friction and the disk friction are practically the same for all of them. In addition the hydraulic friction in the narrow impellers will be greater than that in the larger ones. It is therefore evident that the efficiency of the high-capacity impellers will be much greater

Specific Speed of Centrifugal Pumps.—The specific speed factor for centrifugal pumps is as useful as that for hydraulic turbines. By it, we can at once determine the conditions that are possible for a pump of existing design, and can also select the most suitable combination of factors for a proposed pump for any case. It also serves to classify pump impellers as to type in the same way that it indicates the type of turbine runner.

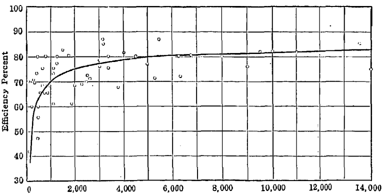

Fig. 148.—Efficiency as a function of capacity.

Thus a low value of the specific speed indicates a narrow impeller of large diameter, while the reverse is true for a high value.

For the centrifugal pump, however, it is more convenient to use a different form for this factor than for the turbine. We are not primarily concerned with the power required to drive a pump, but have our attention centered first upon its capacity. But since the capacity and power are directly related, it is seen that we are merely expressing the specific speed in different units. Since, as in the case of the turbine, $q = K_1D^2\sqrt{h}$, and $N = 1840\phi\sqrt{h}/D$, we may eliminate D between the two equations and obtain

$$N_s' = 1840\phi\sqrt{K_1} = \frac{N\sqrt{q}}{} \qquad (83)$$

Since 448 G.P.M. = 1 cu. ft. per second, it may be seen that $N_s = 21.2N'_s$.

For a single impeller, values of N_s ordinarily range from 500 to 8000. This latter figure has been greatly exceeded in a few cases of special types. Just as in the case of the turbine, the efficiency may be expressed as a function of the specific speed, as is shown in Fig. 149.

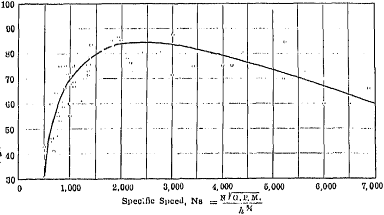

FIG. 149.—Efficiency as a function of specific speed.

171. Conditions of Service.—Centrifugal pumps are used for lifting water to all heights from a few feet to several thousand. Several pumps have been built to work against a head of 2000 ft., though these are all multi-stage pumps. The usual head per stage is not more than 100 to 200 ft., though this figure has been exceeded in numerous instances.

The capacities of centrifugal pumps ranges all the way from very small values up to 300 cu. ft. per second or 134,500 gal. per minute. Rotative speeds range ordinarily from 30 to 3000 r.p.m. according to circumstances. All the above figures are for ordinary practice, and are not meant to be the limiting values

diameter of impeller for the same capacity.

Water leakage from the discharge to the suction side is minimized by the use of clearance rings, as in the case of turbines, and sometimes labyrinth rings are used so as to provide a more tortuous passage for the leakage water. The leakage of air along the shaft on the suction side should be prevented by a water seal in addition to the usual packing.

The end thrust is taken care of by a thrust bearing, by symmetrical construction, as in the case of the double suction pump or a multi-stage pump with impellers set back to back, or by use of an automatic hydraulic balancing piston. The majority of multi-stage pumps are built with the impellers all arranged the same way in the case as this permits the most direct flow from one impeller to the next and also simplifies the mechanical construction.

173. QUESTIONS AND PROBLEMS

1. Why is the centrifugal pump so called? How does the pressure and velocity of the water vary as it flows through such a pump?

2. What classes of centrifugal pumps are there, and how do they differ?

3. What is the difference between the head imparted to the water and the head developed by the pump? How is the latter measured in a test?

4. What are the important hydraulic losses in the centrifugal pump? What is meant by the head of impending delivery? What is its approximate value?

5. How may the head vary with the rate of discharge, the speed being constant? Why is this?

6. If a given centrifugal pump is run at a different speed how will the head, rate of discharge, power, and efficiency vary, assuming that the conditions are such that ϕ is constant?

7. Why is the efficiency of a centrigufal pump a function of its capacity?

8. What is meant by the specific speed of a centrifugal pump? What is the use of such a factor?

9. The diameter of the impeller of a single-stage centrifugal pump is 6 in. If it runs at 2000 r.p.m., what will be the approximate value of the shut-off head and the head for the rate of discharge corresponding to maximum efficiency? *Ans.* 42.5 ft. and 30 ft.

10. What may be the maximum and minimum limits of the capacity of

speed? *Ans.* 1045 r.p.m.

14. Compute the specific speed for the pumps in problems (12) and (13).

15. Compute the factors by which $u_2^2/2g$ must be multiplied to give the shut-off head and the head for highest efficiency for the pumps whose tests are given in Appendix C, Tables 14 and 15.

Let the relation between instantaneous speed and time be represented by the curve shown in the figure. Let

N = r.p.m.
t = seconds.
s = length of subnormals in inches.
x = distance in inches.
y = distance in inches.
m = seconds per inch.
n = r.p.m. per inch.
I = moment of inertia of the rotating mass in ft.-lb. sec.^{2} units.

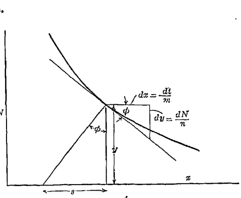

Fig. 150.

Thus $y = N/n$ and $x = t/m$

ω = radians per second $= 2\pi N/60$. $d\omega/dt = (2\pi/60)dN/dt$.

From mechanics, Torque $= Id\omega/dt$. Power $= I\omega d\omega/dt$.

$$\text{Power} = (2\pi/60)^2 INdN/dt$$

Tan $\phi = dy/dx = (dN/n) \div (dt/m)$.

But also tan $\phi = s/y$. Equating these two, $dN/dt = ns/my$. Thus

$$\text{Power} = (2\pi/60)^2(n^2s/m)I.$$

by direct experiment.

STREAM LINES IN CURVED CHANNELS

The following theory is based upon certain assumptions which are only approximately realized in practice, but yet there are many cases which approach these conditions so closely that the methods here given may be successfully applied.[1] Assume that across any section, such as AB in Fig. 151, the total head is constant. This will be true if all particles of water, coming from some source, have lost equal amounts of energy en route and thus all reach the section AB with an equal store of energy. Actually some particles of water may have lost more than others. But

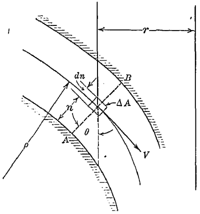

FIG. 151.

if it be assumed that the total head across the section is constant, it follows that, if the pressure is higher at any point, the velocity will be lower than at some other point and vice versa. Owing to centrifugal action, the pressure at B will be greater than that at A and hence the velocity will decrease along the line from A to B, the line AB being normal to the stream lines.

In Fig. 151, let us consider an elementary volume of water

pressure on the two forces and $w\Delta A dn \cos\alpha$ as the component of gravity. The normal acceleration is V^2/ρ. Hence we may apply the proposition that force equals mass times acceleration and obtain

$$dp\Delta A + w\Delta A dn \cos\alpha = (w\Delta A dn/g)(V^2/\rho)$$

Letting dz represent the change in elevation corresponding to dn, we have $dz = dn \cos\alpha$. Thus from the above we may write

$$\frac{gdp}{wdn} + \frac{gdz}{wdn} = \frac{V^2}{\rho} \tag{85}$$

Since $H = \frac{p}{w} + z + \frac{V^2}{2g} =$ constant along line AB, we may differentiate with respect to n and obtain

$$\frac{dH}{dn} = \frac{dp}{wdn} + \frac{dz}{dn} + \frac{2VdV}{2gdn} = 0$$

And from this we may write

$$\frac{gdp}{wdn} + \frac{gdz}{dn} + \frac{VdV}{dn} = 0 \tag{86}$$

Combining equations (85) and (86) we obtain

$$\frac{V^2}{\rho} + \frac{VdV}{dn} = 0$$

This may be written as

$$\frac{dV}{V} = -\frac{dn}{\rho}$$

Integrating

$$\log_e \frac{V}{V_A} = -\int_o^n \frac{dn}{\rho}$$

$$\frac{V}{V_A} = e^{-\int_o^n \frac{dn}{\rho}}$$

$$V = \frac{V_A}{e^{+\int_o^n \frac{dn}{\rho}}} = \frac{V_A}{e^{Y_1}} \tag{87}$$

In the above V_A is the velocity

e crossing AB could be determined.

The solution of the problem from this point depends upon the riation in the cross-section perpendicular to the plane of the per. The remainder of the discussion will be confined to the e where the boundary walls are planes passing through an s of rotation, as shown. The thickness of the elementary

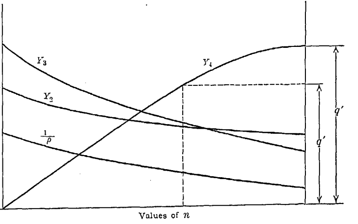

FIG. 152.

ume perpendicular to the plane of the paper will be $r\Delta\theta$. e rate of discharge through the channel between the wall at and the stream line at distance n will be

$$\Delta q' = \int_o^n r\Delta\ \theta\ .\ dn\ .\ V.$$

the entire circumference around the axis from which r is asured, we may substitute 2π for $\Delta\theta$, and, inserting the value V given by equation (87), we have

$$q' = 2\pi V_A \int_o^n \frac{r}{e^{Y_1}} dn = 2\pi V_A \int_o^n Y_3 dn \qquad (88)$$

sponding values of n determined from the Y_4 curve. This fixes the division points for the stream lines along the section AB. A similar procedure may be gone through with for any other sections. If a considerable change is effected in the tentative stream lines first assumed, this may be repeated for the corrected

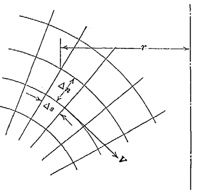

FIG. 153.

set and so on. However extreme accuracy is not warranted, so that a reasonable approximation is quite sufficient.

As a final check on the work it may be noted that if a series of normal lines be drawn, as in Fig. 153,

$$\frac{\Delta n \times r}{\Delta s} = \text{constant} \tag{89}$$

The proof of this is that if two flow lines are spaced dn apart, we may write $\Delta s_1/\Delta s_2 = \rho/(\rho + dn)$. From the preceding treatment, we have $dV/V = -dn/\rho$, from which may be obtained $(V + dV)/V = V_1/V_2 = (\rho - dn)/\rho$. Multiplying both numerator and denominator of the last term by $(\rho + dn)$ and dropping differentials of the second order, we have

$$\frac{V_1}{V_2} = \frac{\Delta s_2}{\Delta s_1} \text{ or } V\Delta s = \text{constant} \tag{90}$$

$$q' = \Delta n r \Delta \theta V = \Delta n r \Delta \theta \times \frac{\text{constant}}{\Delta s}, \text{ since } V\Delta s = \text{constant}.$$

If all stream lines are so spaced as to subdivide the total flow into equal parts, we have for the entire channel $\Delta n.r/\Delta s =$ constant.

In the profile views of the turbine runner are shown only the circular projections of the true stream lines. The application of the preceding theory to this case is open to some uncertainty, but the theory should apply rather closely to the stream lines from the draft tube up to the discharge edge of the runner, since these lines should be in the plane of the paper. The principal object of the procedure is to determine the division points along the outflow edge and the direction of the stream lines at these points, and any uncertainty as to the stream lines within the runner will have little effect upon this. Hence the method is acceptable.

APPENDIX C

TEST DATA

The following data will supply material from which a number of curves may be constructed. Most of it will be found suitable for plotting characteristic curves, if desired.

Tables 1 to 5 inclusive are Holyoke tests of five reaction turbines of different types, taken from "Characteristics of Modern Hydraulic Turbines" by C. W. Larner in *Trans. A. S. C. E.*, Vol. LXVI, p. 306. Table 6 contains the results of the test of an I. P. Morris turbine in the hydro-electric plant of Cornell University.

Tables 7 to 11 inclusive are tests of the same Pelton-Doble tangential water wheel under widely different heads. These were made under the direction of the author by F. W. Hoyt and H. H. Elmendorf, seniors in Sibley College. In general they confirm the conclusions in Art. 103. Within reasonable limits the characteristic curve is about the same regardless of the head under which the test was made. The results show that the efficiency increases rather rapidly as the head is increased from very low values, but, as the effect of mechanical losses becomes relatively less for the higher heads, the efficiency increases but slightly thereafter. It might be expected that the efficiency would approach a certain value as a limit as the head was indefinitely increased, provided the bearings were adapted to the higher speeds. Such might be the case if it were not for another factor. The absolute velocity of discharge, V_2, varies as the square root of the head. For low heads the water discharged from the buckets strikes the case and falls into the tail race without interfering with the wheel. For high heads it was observed that the water was deflected back from the case with sufficient velocity to strike the wheel and thus to greatly increase the values of friction and windage over the values given in Table 12, where no water was present. The head at which this interference began to take place

Some test data taken for the Pelton Water Wheel Co. by the J. G. White Co. will be found in Table 13. The results of tests on two centrifugal pumps of widely different types are given in Tables 14 and 15.

In the construction of characteristic curves, the following method has been found to be very convenient. Construct curves between efficiency and speed under 1-ft. head for the various gate openings. For any given efficiency the speeds for the different gates can be obtained from these and the points thus dermined location on the characteristic curve. The iso-efficiency curves may be drawn through these points, thus eliminating the necessity of interpolation. Smooth efficiency curves, however, should be drawn, since very slight errors in data appear magnified on the characteristic curve.

Company Turbine Wheel, No. 1795

Date, February 18 and 19, 1909. Case No. 1794

Wheel supported by ball-bearing step. Swing-gate. Conical draft-tube

Number of experiment	Porportional part of: Percentage of full opening of speed-gate	Porportional part of: Percentage of full discharge of wheel	Head acting on wheel, in feet	Duration of experiment, in minutes	Revolutions of wheel per minute	Quantity of water discharged by wheel, in cubic feet per second	Horse-power developed by wheel	Percentage of efficiency of wheel
69	1.000	1.024	17.23	4	112.50	34.45	46.84	69.50
67	1.000	1.000	17.50	3	152.67	34.22	55.80	82.16
68	1.000	1.008	17.57	2	163.00	34.05	57.31	83.98
66	1.000	1.005	17.46	3	164.00	34.05	56.90	84.40
65	1.000	0.992	17.47	3	172.67	33.00	55.92	84.00
64	1.000	0.979	17.48	3	181.33	33.17	54.53	82.93
63	1.000	0.965	17.46	4	189.25	32.67	52.53	81.21
62	1.000	0.946	17.47	4	213.00	32.00	49.27	77.57
61	1.000	0.688	17.50	4	251.50	23.39	34.91	74.81
60	1.000	0.631	17.86	4	208.00	21.62	0.00	0.00
59	0.889	0.933	17.50	3	113.67	31.63	45.23	72.04
58	0.889	0.934	17.52	3	135.67	31.70	50.84	80.72
57	0.889	0.931	17.54	3	146.33	31.60	52.80	84.01
56	0.889	0.925	17.53	3	153.67	31.40	53.32	85.41
55	0.889	0.922	17.44	4	156.50	31.21	52.85	85.62
54	0.889	0.916	17.45	3	160.33	31.00	52.66	85.84
53	0.889	0.909	17.46	3	164.00	30.79	52.35	85.87
52	0.889	0.903	17.47	4	168.00	30.59	52.07	85.92
51	0.889	0.898	17.48	4	172.75	30.42	51.95	86.14
50	0.889	0.893	17.48	4	176.75	30.25	51.52	85.91
49	0.889	0.885	17.49	4	184.00	30.05	51.08	85.69
48	0.889	0.864	17.53	4	206.50	29.31	47.77	81.08
47	0.889	0.705	17.50	4	244.75	27.02	33.97	63.02
46	0.889	0.572	17.91	4	208.75	19.61	0.00	0.00
45	0.741	0.800	17.57	4	106.00	27.48	39.23	71.65
44	0.741	0.801	17.57	4	139.50	27.20	45.18	83.35
43	0.741	0.791	17.55	3	152.00	26.85	45.71	85.53
41	0.741	0.784	17.57	4	159.75	26.63	45.82	86.35
40	0.741	0.780	17.60	4	166.25	26.52	46.15	87.18
42	0.741	0.777	17.56	4	171.50	26.40	46.02	87.53
39	0.741	0.770	17.64	4	180.75	26.22	45.90	87.68
38	0.741	0.758	17.67	4	191.50	25.82	44.30	85.61
37	0.741	0.727	17.73	3	210.67	24.81	38.99	78.15
36	0.741	0.693	17.70	3	231.33	23.08	32.11	67.20
35	0.741	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Nun me	Perc full spe	Perc full of v	Hea in f	Dur me	Rev per	Qua dis in sec	Hor ope	Perc cie
28	0.593	0.622	17.91	3	173.00	21.33	36.82	84.98
27	0.593	0.606	17.99	3	188.00	20.82	34.79	81.90
26	0.593	0.537	18.16	4	239.00	18.54	22.11	57.92
25	0.593	0.406	18.05	3	281.00	13.98	0.00	0.00
24	0.444	0.505	17.91	3	94.00	17.32	23.92	67.99
22	0.444	0.489	17.92	2	130.00	16.78	27.06	79.36
21	0.444	0.486	17.94	3	136.00	16.70	27.68	81.48
23	0.444	0.486	17.92	4	139.25	16.66	27.70	81.82
20	0.444	0.481	17.94	3	145.33	16.50	27.57	82.12
19	0.444	0.472	17.94	4	153.00	16.21	26.90	81.56
18	0.444	0.465	17.95	5	161.40	15.98	26.13	80.34
17	0.444	0.450	17.99	4	178.00	15.46	24.71	78.32
16	0.444	0.402	18.04	4	231.50	13.85	16.07	56.70
15	0.444	0.325	18.17	4	281.25	11.22	0.00	0.00
14	0.296	0.325	18.10	3	80.67	11.22	15.35	66.32
13	0.296	0.320	18.21	3	107.00	11.06	16.58	72.60
11	0.296	0.317	18.23	3	114.67	10.97	16.98	74.85
12	0.296	0.316	18.22	3	118.33	10.94	16.97	75.07
10	0.296	0.313	18.24	3	122.67	10.82	17.03	76.07
9	0.296	0.308	18.25	4	130.75	10.68	16.94	76.62
8	0.296	0.304	18.26	4	137.75	10.54	16.57	75.91
7	0.296	0.300	18.30	4	146.50	10.41	16.27	75.29
6	0.296	0.295	18.31	4	154.75	10.23	15.76	74.15
5	0.296	0.291	18.32	4	164.00	10.10	15.17	72.31
4	0.296	0.283	18.33	5	181.20	9.82	14.25	69.81
3	0.296	0.276	18.36	4	191.25	9.58	13.27	66.53
2	0.296	0.266	18.33	4	208.00	9.23	11.56	60.18
1	0.296	0.225	18.37	4	261.00	7.83	0.00	0.00

NOTE.—For experiments Nos. 1, 15, 25, 35, 46 and 60, the jacket was loose.

During the above experiments, the weight of the dynamometer, and of that portion of the shaft which was above the lowest coupling was 1,300 lb.

With the flume empty, a strain of 1.0 lb., applied at a distance of 2.4 ft. from the center of the shaft, sufficed to start the wheel.

Number of exp[...] ment	Percentage of full opening of speed-gate	Percentage of full discharge of wheel	Head acting on w[...] in feet	Duration of ex[...] ment, in minut[...]	Revolutions of w[...] per minute	Quantity of w[...] discharged by w[...] in cubic feet [...] second	Horse-power d[...] oped by wheel	Percentage of [...] ciency of wheel
95	1.077	0.071	17.11	4	153.00	97.00	125.20	66.52
94	1.077	1.017	16.97	3	199.67	101.16	147.66	75.84
93	1.077	1.030	16.94	3	224.33	102.98	156.38	79.04
92	1.077	1.053	16.89	3	239.33	104.50	159.58	79.72
91	1.077	1.061	16.87	3	247.33	105.22	161.17	80.06
89	1.077	1.068	16.81	3	253.67	105.70	161.45	80.12
90	1.077	1.072	16.80	3	259.00	106.08	161.71	80.01
88	1.077	1.070	16.82	4	267.50	106.82	162.15	79.58
87	1.077	1.026	17.05	3	204.67	102.27	125.03	63.23
86	1.000	0.913	17.28	3	147.00	91.65	120.20	66.98
85	1.000	0.957	17.16	2	190.50	95.70	144.34	77.50
84	1.000	0.972	17.14	3	211.67	97.10	152.69	80.89
83	1.000	0.981	17.13	3	225.00	98.03	156.85	82.36
82	1.000	0.990	17.11	3	232.67	98.90	157.96	82.31
80	1.000	0.996	17.09	4	240.25	99.43	160.19	83.13
81	1.000	1.003	17.07	3	247.33	100.07	161.17	83.19
79	1.000	1.004	17.07	4	262.25	100.14	160.55	82.82
78	1.000	1.001	17.13	4	259.00	100.00	157.00	80.82
77	1.000	0.983	17.22	3	268.33	98.43	146.39	76.15
76	1.000	0.911	17.47	4	293.50	91.96	106.75	58.59
106	0.923	0.868	17.32	4	143.25	87.24	115.49	67.30
105	0.923	0.899	17.24	5	176.00	90.12	135.49	76.90
104	0.923	0.920	17.15	4	201.00	91.96	146.21	81.74
101	0.923	0.931	16.93	5	213.20	92.52	148.62	83.66
100	0.923	0.936	16.93	6	220.67	92.96	150.48	84.31
99	0.923	0.942	16.91	4	227.25	93.51	151.52	84.50
102	0.923	0.945	16.93	4	232.00	93.82	151.88	84.31
103	0.923	0.945	17.04	4	235.50	94.14	152.74	83.96
98	0.923	0.945	16.03	4	237.75	93.82	151.32	84.00
97	0.923	0.924	17.02	4	254.50	92.04	138.84	78.15
96	0.923	0.823	17.25	4	288.25	82.47	87.36	54.15
42	0.923	0.870	17.17	3	146.33	87.02	116.20	68.57
41	0.923	0.895	17.10	4	170.00	89.37	130.87	75.51
40	0.923	0.921	17.04	4	202.75	91.80	146.25	82.44
39	0.923	0.928	16.99	3	208.67	92.35	147.99	83.17
35	0.923	0.932	17.03	4	216.25	92.81	150.75	84.10
38	0.923	0.937	16.97	4	220.00	93.20	151.36	84.38
36	0.923	0.939	17.01	4	223.75	93.44	152.58	84.65
34	0.923	0.940	17.02	4	226.25	93.66	150.86	83.45
37	0.923	0.944	16.97	3	238.33	93.90	151.69	83.94
33	0.923	0.921	17.13	4	256.25	92.05	130.80	78.18
32	0.923	0.823	17.27	3	288.00	82.54	87.20	53.99

Number ment	Percentage full open speed-gat	Percentage full disc of wheel	Head acting in feet	Duration ment, in	Revolutions per minu	Quantity discharge in cubic second	Horse-power oped by	Percentage ciency of
31	0.923	0.730	17.50	4	334.75	73.75	0.00	0.00
74	0.846	0.824	17.46	3	158.67	83.15	120.23	73.02
75	0.846	0.836	17.46	3	175.67	84.35	129.91	77.78
72	0.846	0.861	17.34	5	202.20	86.50	143.40	84.30
70	0.846	0.865	17.33	4	209.00	86.95	145.69	85.25
71	0.846	0.868	17.33	3	215.00	87.24	147.27	85.89
73	0.846	0.870	17.34	4	219.25	87.47	148.10	86.15
69	0.846	0.869	17.32	4	221.25	87.32	147.53	86.01
68	0.846	0.866	17.33	4	227.75	87.02	144.06	84.76
67	0.846	0.858	17.36	4	231.75	86.25	140.48	82.73
66	0.846	0.845	17.39	4	243.75	85.11	132.98	79.22
65	0.846	0.828	17.44	4	256.50	83.44	124.39	75.37
64	0.846	0.754	17.59	3	282.00	76.31	85.47	56.15
30	0.769	0.750	17.38	3	141.00	75.52	102.56	68.90
26	0.769	0.766	17.35	3	166.00	77.02	115.72	76.36
27	0.769	0.779	17.32	3	183.00	78.24	124.24	80.84
25	0.769	0.789	17.31	3	194.00	79.25	129.36	83.15
29	0.769	0.793	17.25	4	200.75	79.48	131.42	84.52
28	0.769	0.792	17.26	4	206.00	79.40	131.11	84.36
24	0.769	0.773	17.38	4	226.25	77.82	123.43	80.47
23	0.769	0.735	17.49	3	251.33	74.17	106.64	72.49
22	0.769	0.690	17.56	3	269.67	69.82	81.73	58.78
21	0.769	0.623	17.68	4	323.75	63.27	0.00	0.00
20	0.615	0.617	17.91	4	139.50	63.07	88.79	69.31
16	0.615	0.627	17.79	3	158.33	63.81	95.97	74.55
17	0.615	0.634	17.80	3	171.33	64.55	101.26	77.71
15	0.615	0.638	17.73	4	179.50	64.82	103.37	79.31
18	0.615	0.636	17.77	4	183.00	64.74	103.16	79.07
19	0.615	0.634	17.80	2	188.00	64.61	102.56	78.64
14	0.615	0.627	17.72	4	194.50	63.74	100.21	78.24
13	0.615	0.596	17.77	3	218.67	60.60	92.78	75.07
12	0.615	0.563	17.79	4	243.00	57.20	73.65	63.72
11	0.615	0.519	17.92	4	312.00	53.00	0.00	0.00
10	0.462	0.452	17.34	3	117.33	45.40	56.90	63.73
5	0.462	0.453	17.02	3	136.00	45.65	61.83	70.17
7	0.462	0.461	17.08	4	146.75	46.04	64.94	72.82
6	0.462	0.462	17.05	4	152.00	46.04	66.34	74.52
4	0.462	0.462	16.92	4	155.25	45.90	65.88	74.79
9	0.462	0.459	17.16	3	162.00	45.94	66.78	74.69
8	0.462	0.457	17.15	3	166.67	45.69	65.67	73.90
3	0.462	0.451	16.98	4	172.25	44.83	62.65	72.57
2	0.462	0.432	17.05	4	217.50	43.04	52.74	63.37
1	0.462	0.404	17.18	5	282.40	40.40	0.00	0.00

Number of experiment	Proportional part of: Percentage of full opening of speed-gate	Proportional part of: Percentage of full discharge of wheel	Head acting on wheel, in feet	Duration of experiment, in minutes	Revolutions of wheel per minute	Quantity of water discharged by wheel, in cubic feet per second	Horse-power developed by wheel	Percentage of efficiency of wheel
82	1.000	0.942	17.06	3	140.00	94.80	126.41	68.85
81	1.000	0.950	16.99	3	160.33	95.52	131.16	71.20
80	1.000	0.963	17.00	4	190.50	96.85	144.30	77.28
75	1.000	0.977	17.02	4	210.50	98.34	150.52	79.30
74	1.000	0.984	16.95	4	220.00	98.81	153.31	80.72
70	1.000	0.992	17.01	4	227.25	99.75	155.61	80.87
78	1.000	0.994	16.94	4	229.50	99.75	155.76	81.28
77	1.000	0.997	16.98	4	231.25	100.14	155.55	80.66
79	1.000	1.002	16.90	3	239.67	100.45	156.85	81.47
73	1.000	1.003	16.94	4	254.75	100.70	154.37	79.80
72	1.000	0.871	17.13	3	287.00	87.93	86.96	50.91
71	1.000	0.728	17.44	3	317.67	74.17	0.00	0.00
52	0.923	0.875	17.08	3	131.33	88.14	111.42	65.26
51	0.923	0.896	16.95	3	156.33	89.97	127.89	73.05
50	0.923	0.913	16.93	4	188.25	91.66	142.59	81.02
44	0.923	0.920	16.89	4	201.25	92.20	146.34	82.80
45	0.923	0.927	16.85	4	211.50	92.81	148.67	83.83
46	0.923	0.931	16.86	4	217.00	93.20	149.01	84.12
49	0.023	0.934	16.83	4	220.60	93.42	150.99	84.68
47	0.023	0.936	16.83	4	223.75	93.60	151.86	86.00
48	0.923	0.937	16.83	3	226.33	93.74	150.87	84.32
43	0.923	0.929	16.88	4	241.75	93.13	146.49	82.17
42	0.923	0.788	17.21	4	276.25	79.76	83.70	53.77
41	0.923	0.670	17.44	4	312.50	68.26	0.00	0.00
70	0.846	0.868	17.07	4	192.75	87.47	142.50	84.15
67	0.846	0.875	17.10	4	205.00	88.22	147.83	86.41
68	0.846	0.876	17.06	4	209.50	88.22	148.53	87.02
66	0.846	0.876	17.11	4	212.00	88.37	149.02	86.91
69	0.846	0.877	17.05	3	213.00	88.30	148.43	86.94
65	0.846	0.876	17.13	5	213.80	88.37	147.70	86.03
64	0.846	0.875	17.13	5	215.80	88.30	146.46	85.38
63	0.808	0.810	17.33	4	143.50	82.24	114.78	71.01
62	0.808	0.834	17.25	3	177.00	84.44	135.14	81.81
61	0.808	0.843	17.24	5	194.80	85.35	142.83	85.50
60	0.808	0.847	17.21	3	201.00	85.73	144.94	86.62
50	0.808	0.848	17.10	3	206.00	85.73	146.05	87.30
58	0.808	0.847	17.19	4	208.25	85.65	145.12	86.91
57	0.808	0.815	17.19	3	210.00	85.41	143.80	86.30
56	0.808	0.840	17.22	3	216.67	[illegible]	[illegible]	[illegible]

ment	Percenta[ge of] full ope[ning of] speed-g[ate]	Percenta[ge of] full di[scharge] of whee[l]	Head act[ing] in feet	Duration [of experi]ment, i[n minutes]	Revoluti[ons] per min[ute]	Quantity discha[rged] in cub[ic feet per] second	Horse-po[wer devel]oped by	Percenta[ge of effi]ciency
3	0.808	0.695	17.51	4	264.25	70.97	80.06	56.81
0	0.769	0.755	17.20	4	118.50	76.53	93.35	62.21
0	0.769	0.776	17.21	4	146.50	78.47	110.97	72.46
5	0.769	0.800	17.03	4	174.75	80.40	127.07	81.74
6	0.769	0.806	17.03	4	190.75	81.13	135.24	86.31
7	0.769	0.805	17.10	4	197.25	81.21	137.46	87.28
8	0.769	0.805	17.15	4	200.50	81.28	137.20	86.84
1	0.769	0.803	16.83	4	199.50	80.34	132.98	86.72
3	0.769	0.788	16.93	4	211.75	79.04	128.32	84.55
2	0.769	0.745	16.93	4	232.50	74.72	112.71	78.56
1	0.769	0.660	17.30	4	257.50	67.02	78.02	59.13
0	0.769	0.579	17.60	3	306.00	59.27	0.00	0.00
9	0.615	0.610	17.60	3	100.67	62.60	69.54	55.37
8	0.615	0.621	17.64	3	127.67	63.60	85.10	66.80
5	0.615	0.643	17.60	3	167.33	65.76	104.44	79.57
7	0.615	0.644	17.57	4	175.50	65.83	107.41	81.89
4	0.615	0.643	17.61	4	178.00	65.76	107.86	82.13
6	0.615	0.641	17.50	3	179.67	65.55	106.70	81.60
3	0.615	0.482	17.96	3	206.67	49.84	0.00	0.00
0	0.615	0.634	17.88	4	155.00	65.37	101.44	76.53
1	0.615	0.634	17.80	5	151.40	65.22	98.17	74.56
2	0.615	0.630	17.60	3	160.33	65.50	102.98	78.37
9	0.615	0.644	18.08	5	174.20	66.80	111.80	81.69
8	0.615	0.643	18.11	3	180.00	66.73	112.35	81.07
7	0.615	0.638	18.07	4	183.50	66.19	111.20	81.98
6	0.615	0.636	18.03	4	185.00	65.90	109.86	81.53
5	0.615	0.630	17.96	3	190.00	65.15	109.38	82.43
4	0.615	0.609	17.81	4	201.00	62.66	103.53	81.80
[illegible]	0.615	0.574	17.79	4	216.75	59.07	91.04	77.15
2	0.615	0.553	17.54	5	245.20	56.46	74.29	66.15
1	0.615	0.483	17.71	5	294.60	49.58	0.00	0.00
8	0.462	0.475	17.82	3	104.33	48.95	58.16	58.80
7	0.462	0.483	17.75	4	130.75	49.58	71.98	72.12
6	0.462	0.482	17.76	3	147.00	49.51	73.93	74.14
5	0.462	0.481	17.80	4	156.75	49.45	75.09	76.12
9	0.462	0.477	17.93	2	162.00	49.22	76.57	76.51
0	0.462	0.472	17.97	3	167.00	48.78	75.90	76.35
4	0.462	0.464	17.88	5	173.60	47.80	73.64	75.07
3	0.462	0.436	18.05	4	216.50	45.13	65.66	71.01
2	0.462	0.415	18.08	4	246.25	43.02	44.77	50.75
1	0.462	0.381	18.27	4	280.50	39.75	0.00	0.00

Note.—The jacket was loose for Experiments Nos. 1, 11, 23, 30, 41, and 71.
During the above experiments, the weight of the dynamometer, and of that portion of
... was 2,600 lb.

Number of expe[riment]	Percentage of full opening of speed-gate (part of)	Percentage of full discharge of wheel (part of)	Head acting on whe[el] in feet	Duration of expe[riment], in minutes	Revolutions of whe[el] per minute	Quantity of wa[ter] discharged by whe[el] in cubic feet p[er] second	Horse-power dev[el]oped by wheel
49	1.000	1.049	17.15	3	134.67	79.75	115.81
48	1.000	1.038	17.15	3	147.00	78.95	119.20
47	1.000	1.025	17.19	3	165.67	78.02	123.40
46	1.000	1.024	17.18	2	174.00	77.91	124.34
45	1.000	1.018	17.19	3	178.33	77.52	124.19
44	1.000	1.013	17.19	3	183.33	77.16	124.35
43	1.000	0.012	17.19	4	186.25	77.00	124.07
42	1.000	1.000	17.18	4	189.50	76.82	123.04
41	1.000	1.007	17.10	3	193.00	76.67	123.80
40	1.000	1.006	17.12	4	195.50	76.44	123.13
39	1.000	1.002	17.09	4	200.25	76.10	122.48
38	1.000	0.999	17.10	4	206.25	75.87	122.41
37	1.000	0.997	17.07	3	210.33	75.66	121.01
36	1.000	0.993	17.09	4	219.00	75.38	119.36
35	1.000	0.906	17.24	4	258.75	69.11	78.35
34	1.000	0.744	17.45	4	302.25	57.07	0.00
32	0.883	0.909	17.18	3	120.33	69.23	101.03
31	0.883	0.904	17.20	4	154.25	68.82	111.16
33	0.883	0.901	17.19	4	165.00	68.60	113.01
30	0.883	0.896	17.20	3	172.33	68.25	114.80
29	0.883	0.892	17.21	4	181.50	67.97	116.51
27	0.883	0.888	17.25	3	188.00	67.77	117.27
26	0.883	0.886	17.30	4	194.00	67.71	117.40
28	0.883	0.885	17.23	4	197.25	67.43	117.06
25	0.883	0.883	17.31	4	201.75	67.40	116.07
24	0.883	0.857	17.28	4	213.25	65.42	109.77
23	0.883	0.818	17.31	4	227.00	62.53	96.23
22	0.883	0.761	17.42	4	244.25	58.32	73.96
21	0.883	0.620	17.65	4	291.75	47.83	0.00
69	0.750	0.835	17.28	4	124.75	63.73	91.41
68	0.750	0.839	17.23	4	148.75	63.93	103.59
67	0.750	0.834	17.23	4	168.75	63.60	109.35
64	0.750	0.830	17.24	4	179.25	63.26	110.72
65	0.750	0.829	17.23	4	182.00	63.20	110.77
63	0.750	0.828	17.25	4	186.25	63.12	111.66
66	0.750	0.824	17.23	3	188.67	62.85	110.83
62	0.750	0.821	17.29	4	191.75	62.72	110.32
61	0.750	0.800	17.30	4	197.50	61.81	107.64
60	0.750	0.796	17.34	4	203.25	60.86	104.62
59	0.750	0.768	17.38	4	212.50	58.80	96.52
58	0.750	0.693	17.55	4	237.00	53.31	71.70
57	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Number ment	Percent full op speed-	Percent full of wh	Head in feet	Duration ment,	Revolu per m	Quanti discha in cu second	Horse-p oped b	Percent ciency
20	0.667	0.726	17.27	4	106.25	55.43	70.78	65.20
19	0.667	0.737	17.26	3	148.67	56.25	90.93	82.59
18	0.667	0.739	17.25	4	161.00	56.39	95.55	86.62
17	0.667	0.738	17.24	4	168.50	56.31	96.94	88.05
16	0.667	0.733	17.27	3	173.67	55.92	96.70	88.35
15	0.667	0.722	17.30	4	170.25	55.17	95.52	88.25
14	0.667	0.699	17.34	4	189.50	53.48	91.81	87.30
13	0.667	0.671	17.39	4	201.75	51.37	85.52	84.42
11	0.667	0.508	17.63	4	280.75	39.17	0.00	0.00
8	0.500	0.554	17.82	3	117.00	42.95	60.23	69.30
7	0.500	0.546	17.91	3	135.00	42.46	65.40	75.84
10	0.500	0.548	17.67	3	151.00	42.28	68.58	80.95
6	0.500	0.548	18.05	4	157.50	42.72	71.54	81.80
9	0.500	0.547	17.71	4	157.00	42.28	69.41	81.73
5	0.500	0.539	18.12	5	167.60	42.11	71.05	82.10
4	0.500	0.512	18.13	4	187.00	40.02	67.95	82.58
3	0.500	0.488	18.18	4	213.00	38.21	58.05	73.68
2	0.500	0.460	18.07	4	232.00	35.92	42.15	57.26
1	0.500	0.402	18.20	3	275.00	31.50	0.00	0.00
52	0.333	0.362	18.18	3	96.00	28.35	34.88	59.68
55	0.333	0.361	18.01	3	177.33	28.10	39.08	68.09
51	0.333	0.348	18.19	4	133.00	27.25	40.27	71.64
54	0.333	0.347	18.21	3	139.67	27.20	40.60	72.28
53	0.333	0.340	18.22	3	143.67	26.62	39.15	71.18
56	0.333	0.333	18.06	4	148.00	26.00	37.64	70.60
50	0.333	0.316	18.26	3	201.67	24.82	36.64	71.28

NOTE.—For Experiments Nos. 1, 11, 21, and 57, the jacket was loose.

During the above experiments, the weight of the dynamometer and of that portion of the shaft which was above the lowest coupling was 2,600 lb.

With the flume empty, a strain of 1.0 lb., applied at a distance of 3.2 ft. from the center of the shaft, sufficed to start the wheel.

Number of exp[eri]ment	Percentage of full opening of speed-gate	Percentage of full discharge of wheel	Head acting on wh[eel] in feet	Duraton of exp[eri]ment, in minutes	Revolutions of wh[eel] per minute	Quantity of wa[ter] discharged by wh[eel] in cubic feet [per] second	Horse-power dev[el]oped by wheel	Percentage of e[ffi]ciency of wheel
65	1.000	1.052	17.38	3	112.33	64.87	82.99	64.91
64	1.000	1.040	17.41	3	133.33	64.20	91.24	71.98
63	1.000	1.026	17.43	4	154.25	63.33	96.21	76.85
62	1.000	1.014	17.42	3	168.67	62.60	98.04	79.70
61	1.000	1.009	17.40	3	183.33	62.25	99.92	81.34
60	1.000	1.006	17.42	4	193.25	62.13	100.65	82.00
59	1.000	1.004	17.43	4	201.00	62.01	101.03	82.42
58	1.000	1.002	17.44	4	207.00	61.88	101.54	82.96
56	1.000	1.000	17.35	4	211.00	61.65	100.95	83.22
57	1.000	0.999	17.39	4	216.75	61.65	101.07	83.13
55	1.000	0.998	17.36	4	221.50	61.53	100.60	83.05
54	1.000	0.995	17.38	4	247.50	61.40	97.42	80.50
53	1.000	0.951	17.45	4	266.75	58.80	80.77	69.41
52	1.000	0.730	17.71	4	321.25	45.44	0.00	0.00
51	0.883	0.889	17.48	4	132.25	55.02	79.29	72.60
50	0.883	0.888	17.48	4	147.00	54.91	83.68	76.88
49	0.883	0.885	17.47	4	165.00	54.71	88.93	82.04
48	0.883	0.879	17.39	4	179.25	54.24	91.18	85.24
47	0.883	0.876	17.42	5	189.20	54.11	91.60	85.75
44	0.883	0.873	17.54	4	197.50	54.11	93.29	86.97
43	0.883	0.873	17.40	4	204.25	53.85	92.77	87.30
45	0.883	0.872	17.40	3	209.00	53.79	92.40	87.05
46	0.883	0.871	17.42	4	215.50	53.70	92.66	87.19
42	0.883	0.858	17.40	3	225.00	52.92	88.57	84.81
41	0.883	0.810	17.45	3	248.00	50.06	75.00	75.80
40	0.883	0.609	17.68	3	314.00	37.87	0.00	0.00
75	0.733	0.801	17.76	4	158.50	49.93	81.50	81.13
73	0.733	0.798	17.73	3	183.33	49.70	86.00	86.06
72	0.733	0.797	17.71	3	190.33	49.63	87.66	87.88
71	0.733	0.796	17.72	3	197.00	49.57	88.28	88.62
74	0.733	0.795	17.73	3	199.00	49.50	87.97	88.30
70	0.733	0.793	17.72	3	201.67	49.37	87.93	88.63
69	0.733	0.788	17.70	4	204.50	49.02	86.69	88.10
68	0.733	0.775	17.71	4	213.50	48.27	84.04	86.60
67	0.733	0.722	17.79	3	234.00	45.04	70.85	77.97
66	0.733	0.544	18.03	3	308.67	34.15	0.00	0.00
38	0.667	0.751	17.58	3	171.00	46.58	78.79	84.75
37	0.667	0.750	17.57	4	184.00	46.52	81.34	87.75
39	0.667	0.749	17.56	4	187.00	46.45	81.54	88.14
35	0.667	0.749	17.59	4	185.50	46.45	80.88	87.29

Numb... ment	Percen... full o... speed	Percen... full ... of wh...	Head a... in fee...	Durati... ment,	Revolu... per m...	Quanti... discha... in cu... second...	Horse-... oped ...	Percen... ciency
34	0.667	0.742	17.58	4	192.50	46.02	80.44	87.67
33	0.667	0.731	17.59	4	198.25	45.37	78.04	86.22
32	0.667	0.753	17.70	3	127.07	46.87	68.04	72.32
31	0.667	0.752	17.68	3	144.67	46.76	72.72	77.56
29	0.667	0.752	17.66	4	164.25	46.76	77.59	82.85
30	0.667	0.753	17.68	3	178.67	46.82	81.15	86.44
28	0.667	0.750	17.62	3	189.00	46.57	82.41	88.56
27	0.667	0.741	17.62	3	193.67	46.02	80.93	88.00
26	0.667	0.731	17.64	3	201.00	45.44	79.12	87.04
25	0.667	0.725	17.63	3	204.00	45.05	77.83	86.41
24	0.667	0.709	17.64	3	213.33	44.05	74.93	85.03
23	0.667	0.678	17.70	4	224.50	42.23	67.98	80.19
22	0.667	0.506	17.85	3	303.33	31.61	0.00	0.00
21	0.500	0.566	17.72	2	124.00	35.27	50.31	70.98
20	0.500	0.560	17.74	3	136.33	34.87	52.84	75.32
19	0.500	0.554	17.71	4	146.50	34.50	53.23	76.82
18	0.500	0.543	17.72	4	155.75	33.82	52.82	77.72
17	0.500	0.535	17.74	3	160.00	33.31	52.27	78.00
16	0.500	0.521	17.77	3	176.67	32.47	51.36	78.48
15	0.500	0.507	17.79	3	189.00	31.61	50.36	78.97
14	0.500	0.494	17.83	4	200.25	30.87	48.51	77.71
13	0.500	0.481	17.86	3	214.00	30.08	45.36	74.45
12	0.500	0.464	17.89	4	228.75	29.02	41.56	70.58
11	0.500	0.380	18.00	4	299.50	23.87	0.00	0.00
8	0.333	0.360	18.09	4	112.00	22.65	30.52	65.68
7	0.333	0.351	18.12	3	127.00	22.10	31.53	69.43
6	0.333	0.350	18.13	3	136.00	22.02	32.12	70.95
5	0.333	0.346	18.22	4	141.50	21.84	31.71	70.20
4	0.333	0.340	18.24	4	148.00	21.47	31.37	70.63
3	0.333	0.332	18.31	4	154.75	21.03	30.93	70.82
2	0.333	0.323	18.25	4	165.25	20.42	30.02	71.04
10	0.333	0.307	18.17	4	212.50	19.33	28.31	71.08
9	0.333	0.300	18.15	3	231.33	18.93	25.22	64.72
1	0.333	0.248	18.35	4	287.25	15.74	0.00	0.00

NOTE.—The jacket was loose for Experiments Nos. 1, 11, 22, 40, 52, and 66.

During the above experiments, the weight of the dynamometer and of that portion of the shaft which was above the lowest coupling was 2,600 lb.

With the flume empty, a strain of 0.5 lb., applied at a distance of 3.2 ft. from the center of the shaft, sufficed to start the wheel.

0.067	146.1	7.4	600	143	16.4	13.3
0.067	146.2	6.9	647	0	0	00.0
0.248	144.9	18.7	0	2760	0	00.0
0.248	145.4	16.4	600	1365	156	57.6
0.248	145.7	12.5	845	0	0	00.0
0.476	143.9	27.5	0	5390	0	00.0
0.476	144.3	25.4	600	2760	318	76.3
0.476	145.2	18.8	975	0	0	00.0
0.600	142.8	34.8	0	6550	0	00.0
0.600	143.1	31.8	600	3820	437	84.5
0.600	144.7	23.0	1022	0	0	00.0
0.772	141.8	40.2	0	7520	0	00.0
0.772	141.8	38.8	600	4820	550	88.0
0.772	144.0	26.8	1038	0	0	00.0
1.000	140.6	46.3	0	8130	0	00.0
1.000	140.5	44.5	600	5400	617	87.0
1.000	143.4	32.4	1060	0	0	00.0

[1] For an account of this test see "Investigation of the Performance of Reaction Turbine," Trans. A. S. C. E., Vol. LXXVIII, p. 1270 (1915).

			100	0.32	0.00712	40.0
			150	0.24	0.00802	45.0
			200	0.16	0.00712	40.0
			250	0.06	0.00334	18.8
			280	0.00	0.00000	00.0
2	8.936	0.0300	0	0.95	0.0000	00.0
			100	0.71	0.0158	51.8
			150	0.58	0.0194	63.7
			200	0.43	0.0191	62.8
			250	0.27	0.0150	49.2
			320	0.00	0.0000	00.0
3	8.943	0.0448	0	1.50	0.0000	00.0
			100	1.11	0.0247	54.2
			150	0.90	0.0301	66,0
			200	0.67	0.0298	65.3
			250	0.42	0.0234	51.3
			330	0.00	0.0000	00.0
4	8.951	0.0575	0	1.80	0.0000	00.0
			100	1.38	0.0347	59.3
			150	1.16	0.0388	66.3
			200	0.90	0.0400	68.3
			250	0.60	0.0334	57.0
			340	0.00	0.0000	00.0
5	8.959	0.0670	0	2.00	0.0000	00.0
			100	1.56	0.0347	50.8
			150	1.32	0.0441	64.6
			200	1.06	0.0472	69.2
			250	0.82	0.0457	67.0
			360	0.00	0.0000	00.0
6	8.966	0.0750	0	2.14	0.0000	00.0
			100	1.69	0.0376	49.2
			150	1.46	0.0488	63.8
			200	1.19	0.0526	68.8
			250	0.90	0.0502	65.7
			360	0.00	0.0000	00.0
7.85	8.978	0.0860	0	2.60	0.0000	00.0
			100	1.89	0.0422	48.0
			150	1.66	0.0555	63.2
			200	1.34	0.0596	67.8
			250	1.03	0.0574	65.0
			360	0.00	0.0000	00.0

Wheel under a Constant Pressure Head of 62.5 Ft.

Turns of needle	Head, ft.	Discharge, cu. ft. per sec.	R.p.m.	Brake load, lb.	B.h.p.	Efficiency, per cent.
1	62.52	0.041	100	3.10	0.069	22.8
			200	2.85	0.127	42.0
			300	2.50	0.167	55.2
			400	2.10	0.187	61.8
			450	1.90	0.191	63.0
			500	1.70	0.189	62.5
			600	1.15	0.153	50.7
			700	0.55	0.086	28.4
			775	0.00	0.000	00.0
[illegible]	62.55	0.081	100	6.70	0.149	25.9
			200	6.10	0.276	48.0
			300	5.40	0.361	62.7
			400	4.70	0.419	72.8
			500	3.85	0.429	74.5
			600	3.00	0.400	69.5
			700	1.80	0.284	49.2
			800	0.70	0.125	21.6
			880	0.00	0.000	00.0
	62.58	0.117	100	9.60	0.214	25.6
			200	8.70	0.388	46.5
			300	7.80	0.522	62.5
			400	6.80	0.606	72.6
			500	5.85	0.652	78.0
			600	4.75	0.635	76.0
			700	3.30	0.515	61.7
			800	1.65	0.294	35.2
			930	0.00	0.000	00.0
	62.63	0.150	100	12.30	0.274	25.6
			200	11.25	0.510	47.6
			300	10.05	0.672	62.8
			400	8.85	0.790	73.8
			500	7.80	0.870	81.3
			600	6.45	[illegible]	[illegible]

			200	12.90	0.575	46.4
			300	11.50	0.768	62.0
			400	10.20	0.908	73.3
			500	9.05	1.010	81.5
			600	7.30	0.975	78.7
			700	5.45	0.850	68.6
			800	3.30	0.515	41.6
			900	1.20	0.241	19.4
			970	0.00	0.000	00.0
6	62.74	0.200	100	15.25	0.340	23.8
			200	14.00	0.624	43.7
			300	12.65	0.846	59.5
			400	11.25	1.011	71.0
			500	9.95	1.110	78.0
			600	8.20	1.098	77.0
			700	6.00	0.935	65.6
			800	3.60	0.561	39.4
			900	1.20	0.302	21.2
			985	0.00	0.000	00.0
7.85	62.81	0.230	100	17.40	0.388	23.7
			200	16.10	0.717	43.8
			300	14.70	0.982	60.0
			400	12.80	1.140	69.5
			500	11.10	1.235	75.5
			600	9.30	1.242	76.0
			700	6.90	1.080	66.0
			800	4.20	0.655	40.0
			900	1.85	0.370	22.6
			985	0.00	0.000	00.0

1	130.68	0.062	100	[illegible]	[illegible]	[illegible]
			300	6.2	0.414	44.8
			500	5.2	0.579	62.8
			700	4.0	0.624	67.6
			900	2.2	0.440	47.7
			1100	0.6	0.147	15.9
			1190	0.0	0.000	00.0
2	130.74	0.118	100	14.2	0.316	18.0
			300	12.8	0.855	48.6
			500	10.8	1.201	68.3
			700	8.8	1.372	78.0
			900	6.2	1.242	70.6
			1100	3.2	0.784	44.5
			1300	0.5	0.144	08.2
			1340	0.0	0.000	00.0
3	130.83	0.173	100	21.2	0.472	18.4
			300	18.6	1.242	48.3
			500	15.8	1.760	68.5
			700	12.9	2.010	78.2
			800	11.7	2.080	81.0
			900	10.0	2.002	78.0
			1100	5.4	1.320	51.4
			1300	1.6	0.398	15.5
			1395	0.0	0.000	00.0
4	130.95	0.222	100	26.6	0.592	17.9
			300	24.0	1.602	48.6
			500	20.6	2.295	69.5
			700	16.8	2.620	79.5
			800	15.0	2.670	81.0
			900	12.8	2.660	80.6
			1100	7.6	1.860	56.5
			1300	2.6	0.752	22.8
			1420	0.0	0.000	00.0
5	131.06	0.262	100	31.4	0.700	17.9
			300	28.2	1.884	48.2
			500	24.2	2.695	69.0
			700	19.5	3.040	78.0

			900	15.2	3.040	78.0
			1100	8.8	2.155	55.2
			1300	3.4	0.984	25.2
			1450	0.0	0.000	00.0
6	131.19	0.300	100	34.2	0.762	17.3
			300	31.0	2.070	47.0
			500	26.6	2.960	67.4
			700	21.4	3.335	75.8
			800	19.1	3.400	77.3
			900	16.5	3.300	75.0
			1100	10.2	2.497	56.7
			1300	4.2	1.215	27.6
			1460	0.0	0.000	00.0
7.85	131.40	0.356	100	39.2	0.874	16.4
			300	35.0	2.340	44.0
			500	29.8	3.315	62.2
			700	24.6	3.830	72.0
			800	21.8	3.880	73.0
			900	18.7	3.740	70.3
			1100	11.6	2.838	53.3
			1300	4.8	1.390	26.2
			1460	0.0	0.000	00.0

1	230.2	0.081	100	11.4	0.254	12.0
			300	10.3	0.687	32.4
			500	9.2	1.025	48.4
			700	7.8	1.217	57.5
			800	7.0	1.248	59.0
			900	6.2	1.242	58.7
			1100	4.3	1.054	49.7
			1300	2.0	0.578	27.3
			1440	0.0	0.000	00.0
2	230.3	0.163	100	24.6	0.548	12.8
			300	22.3	1.490	34.9
			500	19.7	2.190	51.2
			700	17.1	2.665	62.4
			900	14.4	2.880	67.5
			1100	11.4	2.795	65.4
			1300	8.0	2.320	54.3
			1500	4.5	1.505	35.2
			1700	1.0	0.378	12.5
				0.0	0.000	00.0
3	230.5	0.231	100	31.3	0.698	11.5
			300	32.7	2.185	36.2
			500	28.3	3.150	52.1
			700	25.6	3.990	66.0
			900	21.6	4.330	71.6
			1100	17.4	4.268	70.6
			1300	12.4	3.588	59.4
			1500	7.0	2.340	38.7
			1700	2.0	0.756	12.5
			1760	0.0	0.000	00.0
4	230.6	0.291	100	45.2	1.005	13.2
			300	41.6	2.780	36.4
			500	37.4	4.160	54.5
			700	33.0	5.145	67.4
			900	28.2	5.650	74.0
			1100	23.1	5.665	74.3
			1300	17.6	5.096	66.7
			1500	9.5	3.180	41.7
			1700	3.9	1.477	19.3
			[illegible]	[illegible]	[illegible]	[illegible]

			[illegible]	[illegible]	[illegible]	[illegible]
			300	49.6	3.315	36.8
			500	44.4	4.940	54.9
			700	38.8	6.050	67.2
			900	32.6	6.530	72.6
			1100	26.6	6.512	72.4
			1300	20.2	5.850	65.0
			1500	12.8	4.280	47.6
			1700	5.4	2.040	22.7
			1880	0.0	0.000	00.0
6	231.1	0.379	100	61.0	1.360	13.6
			300	55.5	3.710	37.2
			500	49.5	5.510	55.4
			700	43.0	6.700	67.3
			900	36.4	7.300	73.4
			1000	33.2	7.390	74.2
			1100	29.7	7.270	73.0
			1300	32.4	6.480	65.0
			1500	14.4	4.800	48.2
			1700	6.4	2.420	24.3
			1890	0.0	0.000	00.0
7.85	231.2	0.434	100	67.2	1.499	13.3
			300	61.8	4.130	36.8
			500	55.4	6.160	54.9
			700	47.8	7.450	66.3
			900	40.0	8.015	71.4
			1000	36.5	8.125	72.3
			1100	32.9	8.063	71.8
			1300	24.8	7.180	64.0
			1500	16.3	5.450	48.5
			1700	7.0	2.550	22.7
			1890	0.0	0.000	00.0

Turns of needle	Head, ft.	Discharge, cu. ft. per sec.	R.p.m.	Brake load, lb.	B.h.p.	Efficiency, per cent.
1	305.1	0.1025	0	18.0	0.00	00.0
			400	15.8	1.41	39.6
			800	12.6	2.24	63.2
			1000	10.8	2.41	67.7
			1200	8.4	2.24	63.2
			1400	6.0	1.87	52.7
			1600	3.2	1.14	32.2
			1800	0.4	0.16	4.5
			1920	0.0	0.00	0.0
2	305.2	0.185	0	35.6	0.00	00.0
			400	30.8	2.74	42.7
			800	24.6	4.38	68.4
			1000	21.0	4.67	72.7
			1200	17.5	4.67	72.7
			1400	13.4	4.18	65.1
			1600	9.1	3.24	50.5
			1800	4.4	1.76	27.4
			2020	0.0	0.00	00.0
3	305.5	0.278	0	52.8	0.00	00.0
			400	45.2	4.02	41.5
			800	36.6	6.52	67.4
			1000	32.0	7.12	73.7
			1200	26.8	7.15	74.0
			1400	20.7	6.45	66.8
			1600	14.4	5.13	53.0
			1800	7.8	3.12	32.2
			2080	0.0	0.00	00.0
4	305.7	0.341	0	62.8	0.00	00.0
			400	53.6	4.77	40.3
			800	43.0	7.65	64.7
			1000	38.4	8.55	72.2
			1200	33.0	8.82	74.5
			1400	26.0	8.10	68.4
			1600	18.6	6.62	55.9
			1800	10.0	4.00	[illegible]

[illegible]	[illegible]	0	72.0	0.00	00.0
		400	62.6	5.57	40.9
		800	52.0	9.27	68.1
		1000	45.8	10.20	75.0
		1200	38.4	10.25	75.3
		1400	30.2	9.40	69.0
		1600	21.0	7.48	55.0
		1800	11.0	4.40	32.4
		2150	00.0	0.00	00.0

12.—FRICTION AND WINDAGE OF 12-INCH PELTON-DOBLE TANGENTIAL WATER WHEEL

[illegible]p.m.	H.p.	R.p.m.	H.p.
[illegible]00	0.0025	800	0.1545
[illegible]00	0.0089	900	0.2190
[illegible]00	0.0146	1000	0.2660
[illegible]00	0.0305	1100	0.3270
[illegible]00	0.0515	1200	0.3910
[illegible]00	0.0746	1300	0.4980
[illegible]00	0.1135	1400	0.5970
		1500	0.7020

3.—TEST OF A PELTON-DOBLE TANGENTIAL WATER WHEEL NEAR FRESNO, CAL.

Static Head = 1403.45 ft.

[illegible]d,	Discharge, cu. ft. per sec.	H.p. input	B.h.p.	Efficiency, per cent.
[illegible].77	17.50	2790	2075	74.4
[illegible].25	22.10	3510	2767	78.7
[illegible].56	27.00	4280	3450	80.7
[illegible].15	31.90	5050	4120	81.7
[illegible].80	36.70	5800	4765	82.2
[illegible].65	42.00	6630	5480	82.7
[illegible].90	54.00	8475	6825	80.6

1700 r.p.m.

arge, cu. ft. per sec.	Head, ft.	B.h.p.	Efficiency, per cent.
0.000	248.5	9.13	00.0
0.049	248.6	10.12	13.7
0.112	254.6	11.78	27.6
0.155	257.5	12.70	35.8
0.236	264.1	15.08	47.1
0.348	248.8	18.15	54.3
0.429	225.0	20.06	55.0
0.494	192.2	21.60	50.0
0.531	157.3	22.00	43.2
0.573	73.1	21.25	22.4
0.578	47.1	20.60	15.0
0.580	26.3	20.15	8.6

15.—Test of a 6-Inch Single-Stage DeLaval Centrifugal Pump at Cornell University

By R. L. Daugherty

te Type. Diameter of Impeller = 9.11 inches. Speed 1700 r.p.m.

arge, cu. ft. per sec.	Head, ft.	B.h.p.	Efficiency, per cent.
0.000	68.5	4.3	00.0
0.068	68.4	4.5	11.7
0.188	69.6	5.2	28.6
0.320	69.3	6.0	42.0
0.606	69.2	7.8	61.0
0.873	65.8	9.3	70.2
1.063	62.7	10.3	73.5
1.315	55.7	11.3	73.7
1.632	47.3	12.0	73.2
1.968	35.7	11.8	67.7
2.090	28.1	11.5	58.0
2.240	22.3	11.2	50.7

A

B

C

D

E

F